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DECOMPOSITION APPROACH TO THE NEWTON- RAPHSON BASED HYDRO-THERMAL OPTIMAL POWER FLOW SOLUTION

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ABSTRACT

This paper presents a decomposition approach to the hydro-thermal optimal power flow problem to address the mathematical difficulty due to large number of variables and constraints associated with the problem. The decomposition of the problem is achieved by dividing the optimization period under consideration into hourly time intervals. The optimization problem for each time interval is first solved using the Newton-Raphson based solution technique. The results associated with each hydro plant for every hour are then used to adjust the water worth value for each plant. This procedure is repeated until the available water is optimally utilized. The proposed solution procedure is implemented using MATLAB version 7.14 software. The performance of the proposed algorithm is tested on IEEE 5-bus, 30-bus and 57-bus networks. The simulation results have shown the effectiveness of the proposed algorithm. The performance index of each hydro station has been measured using the average hydro plant energy.

1. INTRODUCTION

Successful planning and operation of the power system are the key goals of the power system engineering. These goals involve meeting the demand of electricity of the consumers at every time instance, minimizing the environmental impact of operation of power system, ensuring safety of both personnel and equipment. However, these goals must be economically accomplished [1]. One way to economically accomplish these goals is through economic dispatch.

Economic dispatch is defined as the process of scheduling individual generating units in the power system, so that the system load is supplied entirely, and most economically [2]. Economic scheduling of the power system that contains both hydro and thermal stations involves the optimization of one or more objective functions with due consideration for the usage of limited water available for hydro generation [3-5]. This is generally referred to as hydro-thermal optimal scheduling problem. This problem can be classified into two. The first, which does not consider the power flow equations and the second, which considers the power flow equations and it is generally referred to as the hydro-thermal optimal power flow (HTOPF) problem.

The hydro-thermal optimal scheduling problem without the consideration of power flow equations has been solved in the past with various approaches. These approaches are either deterministic or heuristic [6]. Deterministic methods take into consideration the analytical properties of the problem to create a sequence of points that converge to a global optimal solution. A major drawback to this approach is excessive reliance on good initial variables and the involvement of derivatives. On the other hand, heuristic methods have been found to be more flexible and efficient than deterministic approaches, however, the quality of the obtained solution cannot be guaranteed [6, 7]. Among the deterministic techniques are the base load procedure [1], variational calculus [8], coordination equation [9], dynamic programming [10, 11], Pontryagin maximum principle [12], peak shaving [9], Langrangian relaxation method [13, 14], Newton's method [15], nonlinear optimization method [16] and mixed-integer programming methods [17]. The heuristic methods include the genetic algorithms [18], evaporation rate–based water cycle algorithm [19], particle swarm optimization [20], evolutionary programming [21], clonal selection algorithm [22] and so on.

The HTOPF problem has been solved in the past with the progressive optimality algorithm, Newton iteration method, interior point method, linear and non-linear programming, a hybrid of genetic algorithm and Lambda iteration technique and so on.

Progressive optimality algorithm is one of the methods used to solve the HTOPF problem in the early 1980s. This approach performed well when applied to handle the optimal schedule of generation in power systems that consist of cascaded hydro plants with time delay, head variation and constraints imposed due to equipment ratings and operating conditions of the power systems [23].

The Newton formulation appears to be as fundamental and effective for solving the HTOPF problem as it is for optimal power flow (OPF) and power flow. One of the earliest applications of the Newton's approach to optimization of power systems operation was on all-thermal stations with non-separable objective function [24]. The results obtained were quite promising. Other works later extended the approach to the HTOPF [3-5, 25, 26]. Some of these works used the fuel cost as the objective function [3, 25] and some other used the transmission loss as the objective function [26]. Some formulated the problem as a multi-objective function with fuel cost and transmission loss as the objective functions [4]. Most of these works applied their solution to various test systems using load curve with two to three discrete time intervals.

The reasons for the choice of using Newton-Raphson method for solving non-linear power equations that describe the power systems are [3, 4, 6];

(a) The solution technique is efficient for large-scale power system analysis
(b) The method has improved, reliable and favorable convergence characteristics over other methods,
(c) The Newton method has surpassed other methods in the aspects of memory demand and computing speed.
(d) The numbers of iteration required for solution is independent of the size of the problem.

Its major drawback is that convergence characteristics are highly dependent on the value of the initial conditions.

Interior point method involves finding improved search directions strictly in the interior of the feasible space [27]. This method has proven to be a promising alternative for the solution of power system optimization problems. This method has been adopted in some HTOPF works to solve the thermal sub-problem while the hydro sub-problem is solved by the analytical method [28] or the network programming method [29, 30]. However, the thermal and hydro sub-problems in HTOPF problem have also been handled simultaneously using interior point method [31].

Linear programming formulation involves the linearization of objective function as well as constraints with non-negative variables [32, 33]. On the other hand, nonlinear programming technique uses nonlinear objective function and constraints. Linear and nonlinear programming methods are capable of handling the thermal sub-problem of the HTOPF problem. Linear programming has been used to achieve a near optimal solution that can be used to start the non-linear programming technique [34].

In a bid to look for modern approach of solving the HTOPF problem, researchers are now adopting a combination of the deterministic and heuristic technique of solving optimization problem. In a recent work [35], the hydro sub-problem has been solved with the genetic algorithm approach while the thermal sub-problem is solved with the lambda iteration technique. This approach was reported to have a near global optimum solution.

Due to the maximization of available water energy during a specified time interval associated with the HTOPF problem, it has usually been solved by decomposing the problem into two stages [6, 36]. The hydro sub-problem is firstly solved to define the hydro energy generation pattern. Based on the decision of the first stage, the thermal sub-problem is solved. This approach may not yield optimal results since usually, the real power output of the hydro stations are fixed during the thermal generation dispatch. However, other methods solved both the hydro and thermal subproblems simultaneously. These methods either utilize the Newton Raphson (N-R) [3, 4, 25] or the interior point [31]. The results presented in those works proved the success of the methods.

The simultaneous consideration of both the hydro and thermal plants constraints in the N-R based HTOPF solution procedure requires a large size of linearized equations [30].The size of these equations increases with system network scale and (or) time interval. This is as a result of the coupled relationship introduced due to water energy constraints. As a result of this, solving these equations directly may cause the solution procedure to be prone to failure as the size of the network increases or as the interval increases. Due to the aforementioned problem, this paper proposes an improved N-R based HTOPF solution technique. The proposed technique decomposes the optimization problem into hourly subproblems to facilitate the solution of the problem. The solution to the subproblem for each hour requires smaller size of equations that only depend on the network scale. These equations are solved with the consideration of all constraints except the water availability constraint. After solving for all the time intervals, the results associated with each hydro plant are used to adjust the water worth value for each plant. This procedure is repeated until the available water is optimally utilized. The proposed procedure has been applied to solve IEEE 5-bus, 30-bus and 57-bus networks. The results presented have verified the effectiveness of the approach to HTOPF problem.

2. PROBLEM DEFINITION

Unlike the OPF which is a one-time solution problem, the additional difficulty introduced by the HTOPF into the OPF is the optimum usage of water over a specified period of time (i.e. one day, one week, one month, one year and so on) [3, 4]. The HTOPF problem can be formulated as follows;

Minimize

$$F = \sum_{t=1}^{T} f(P_j(t)) \tag{1}$$

$$f(P_j(t)) = \sum_{j=1}^{nt} (a_j + b_j P_j(t) + c_j P_j(t)^2) \tag{2}$$

Where F is the total fuel cost of the thermal units for the optimization period; $f(P_j(t))$ is the total fuel cost of thermal units per hour; t refers to the discrete time interval in hour; T is the optimization period under consideration; nt is the total number of thermal stations; a_j, b_j, c_j are the fuel cost coefficients of thermal station j; and $P_j(t)$ is the real power output of thermal generator j.

Equation (1) is subject to these equality constraints:

(a) power balance constraints: In the formulation, the total active and reactive power generations in the system is designed to meet up with the respective demands as indicated in (3).

$$\left.\begin{aligned} \Delta P_i(t) &= P_i(t) + P_{di}(t) - P_{gi}(t) = 0 \\ \Delta Q_i(t) &= Q_i(t) + Q_{di}(t) - Q_{gi}(t) = 0 \end{aligned}\right\rangle \tag{3}$$

Where $P_{di}(t)$ is the active power demands at bus i; $Q_{di}(t)$ is the reactive power demands at bus i; $P_{gi}(t)$ is the scheduled active power generations at bus i (it can either be from the thermal station (i.e. $P_j(t)$) or hydro station (i.e. $P_h(t)$)); $Q_{gi}(t)$ is the scheduled reactive power generations at bus i; $\Delta P_i(t)$ and $\Delta Q_i(t)$ are, respectively, the active and reactive power mismatches at bus i; $P_i(t)$ and $Q_i(t)$ are, respectively, the active and reactive power injections at bus i and are given as:

$$\left.\begin{aligned} P_i(t) &= V_i(t)\sum_{k=1}^{nb} V_k(t)\, Y_{ik}\, cos(\delta_k(t) - \delta_i(t) + \theta_{ik}) \\ Q_i(t) &= -V_i(t)\sum_{k=1}^{nb} V_k(t)\, Y_{ik}\, sin(\delta_k(t) - \delta_i(t) + \theta_{ik}) \end{aligned}\right\rangle \tag{4}$$

Where nb refers to the total number of buses in the system; $V_i(t)$ and $V_k(t)$ are, respectively, the voltage magnitudes at buses i and k; $\delta_i(t)$ and $\delta_k(t)$ are, respectively, the voltage phase angles at buses i and k; Y_{ik} is the magnitude of the admittance of the line connecting buses i and k together; and θ_{ik} is the angle of the admittance of the line connecting buses i and k together.

(b) available water energy constraints: the pre-specified volume of water must be optimally utilized during the optimization period as indicated in (5).

$$q_h - \sum_{t=1}^{T} q(P_h(t)) = 0 \tag{5}$$

$$q(P_h(t)) = \alpha_h + \beta_h P_h(t) + \gamma_h P_h(t)^2 \tag{6}$$

Where q_h is the pre-specified amount of water needed for generation at hydro station h during the optimization period; $q(P_h(t))$ is the volume of water used per hour at hydro station h; α_h, β_h, γ_h are the discharge coefficients of hydro station h; $P_h(t)$ is the real output power of the hydro station h and nh is the total number of hydro stations.

Equation (1) is also subject to these inequality constraints:

$$\left.\begin{aligned} P_{gi}^{min} &\le P_{gi}(t) \le P_{gi}^{max} \\ Q_{gi}^{min} &\le Q_{gi}(t) \le Q_{gi}^{max} \\ V_i^{min} &\le V_i(t) \le V_i^{max} \\ V_k^{min} &\le V_k(t) \le V_k^{max} \end{aligned}\right\} \quad t = 1,2,3,\ldots,T \tag{7}$$

Where superscript *max* and *min* given in (7), respectively, stand for the maximum and minimum limits on the variables.

3. NEWTON-RAPHSON HYDRO-THERMAL OPTIMAL POWER FLOW SOLUTION

The HTOPF problem can be reformulated by augmenting both the power balance constraints during time t and water availability constraints for the optimization interval with the objective function of Equation (1). This is done with the introduction of Lagrange multipliers to cater for the power balance constraints, water worth (or water conversion factor) to cater for the water availability (υ_h) constraints and a penalty function to cater for the inequality constraints [3, 25, 28, 37]. The resulting augmented Lagrangian function is given in (8).

$$L(z,\lambda,\upsilon) = \sum_{t=1}^{T}\left[\begin{array}{l} f(P_j(t)) + \sum_{h=1}^{nh}\upsilon_h q(P_h(t)) \\ + \sum_{i=1}^{nb}\lambda_{pi}(t)\Delta P_i(t) + \sum_{i=1}^{nb}\lambda_{qi}(t)\Delta Q_i(t) \\ + E(h(z),\mu) + G_{qi}(t) \end{array}\right] - \sum_{h=1}^{nh}\upsilon_h q_h \tag{8}$$

Where z represents the state and the control variables of power systems (i.e. real power generation, voltage magnitudes and angles); $\lambda_{pi}(t)$ and $\lambda_{qi}(t)$, respectively, represent the Langrangian multipliers for active and reactive power equations; υ_h is the water worth for hydro unit h; $E(h(z),\mu)$ and $G_{qi}(t)$ represent the penalty functions of the inequality constraint and are respectively given in (9) and (10) [37].

$$E(h(z),\mu) = \begin{cases} \mu[h(z) - \bar{h}] + 0.5c[h(z) - \bar{h}]^2 & \text{if } \mu + c[h(z) - \bar{h}] \ge 0 \\ \mu[h(z) - \underline{h}] + 0.5c[h(z) - \underline{h}]^2 & \text{if } \mu + c[h(z) - \underline{h}] \le 0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$$G_{qi}(t) = \frac{1}{2} S\lambda_{qi}(t)^2 \tag{10}$$

Where $\bar{h}$ and $\underline{h}$ are the upper and lower limits on variables respectively; c is the weighing parameter; and μ is the inequality constraints multiplier; S is a large, positive penalty weighting factor.

Minimizing (8) directly is difficult [28, 30] and its solution procedure is susceptible to failure as the network size and (or) time interval increases. On the other hand, this work decomposes (8) into hourly sub-equations as given in (11). These equations are minimized for every hour and an iterative adjustment strategy is subsequently developed to cater for υ, in other to satisfy the condition given in (5). The procedure involved in the proposed approach is given in the following section.

4. PROPOSED DECOMPOSITION APPROACH

The aim of this approach is to relax the water availability constraint in (8). Doing this requires having T Lagrangian sub-equations. For example, if the optimization period under consideration is 24-hour, the proposed approach requires 24 Lagrangian equations. Each equation is minimized separately and the results for all the equations are used to update the water worth value. This procedure continues until the water worth value tracks the available water.

4.1 The Decomposed Lagrangian Function

The decomposed Lagrangian function for the proposed approach is given as:

$$L_t(z,\lambda,\upsilon) = f(P_j(t)) + \sum_{h=1}^{nh}\upsilon_h q(P_h(t)) + \sum_{i=1}^{nb}\lambda_{pi}(t)\Delta P_i(t) + \sum_{i=1}^{nb}\lambda_{qi}(t)\Delta Q_i(t) + E(h(z),\mu) + G_{qi}(t) \quad t=1,2,3,\ldots,T \tag{11}$$

Unlike (8) where the water energy constraint is included, the new function does not. The water energy constraint will however be considered after minimizing all the T Lagrangian sub-equations. The procedure for minimization is given below. It should be noted that the penalty functions of (9) and (10) are only necessary in (11) for limits enforcement. How inequality constraints are handled is discussed later.

The Karush-Kuhn-Tucker (KKT) condition for optimality of (11) is given as:

$$\frac{\partial L_t}{\partial z} = \nabla_z L_t = \begin{bmatrix} \frac{\partial L_t}{\partial \delta_i(t)} \\ \frac{\partial L_t}{\partial \delta_k(t)} \\ \frac{\partial L_t}{\partial V_i(t)} \\ \frac{\partial L_t}{\partial V_k(t)} \\ \frac{\partial L_t}{\partial P_{gi}(t)} \end{bmatrix} = 0 \tag{12}$$

$$\frac{\partial L_t}{\partial \lambda} = \nabla_\lambda L_t = \begin{bmatrix} \frac{\partial L_t}{\partial \lambda_{pi}(t)} & \frac{\partial L_t}{\partial \lambda_{qi}(t)} \end{bmatrix}^T = 0 \tag{13}$$

The solution of (12) and (13) by Newton's approach requires linearizing these equations to give:

$$\begin{bmatrix} \frac{\partial^2 L_t}{\partial z^2} & \frac{\partial^2 L_t}{\partial z \partial \lambda} \\ \frac{\partial^2 L_t}{\partial \lambda \partial z} & \frac{\partial^2 L_t}{\partial \lambda^2} \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} \nabla_z L_t \\ \nabla_\lambda L_t \end{bmatrix} \tag{14}$$

Where $\begin{bmatrix} \frac{\partial^2 L_t}{\partial z^2} & \frac{\partial^2 L_t}{\partial z \partial \lambda} \\ \frac{\partial^2 L_t}{\partial \lambda \partial z} & \frac{\partial^2 L_t}{\partial \lambda^2} \end{bmatrix}$ is the Hessian matrix, $\begin{bmatrix} \nabla_z L_t & \nabla_\lambda L_t \end{bmatrix}^T$ is the gradient vector, Δz and $\Delta\lambda$ are, respectively, the increments or decrements on control and state variables and the Langrangian multipliers.

Equation (14) is solved iteratively and for every iteration, the variables (z and λ) are updated using the equation given below:

$$\left. \begin{aligned} z^{new} &= z^{old} - \Delta z \\ \lambda^{new} &= \lambda^{old} - \Delta\lambda \end{aligned} \right\rangle \tag{15}$$

A solution is reached if the gradient vectors of (12) and (13) are within a tolerance margin of 10^{-4}.

If any of the variables listed in (7) is outside the allowable limits, the inequality constraint is enforced as described in the next section.

4.2 Handling of Inequality Constraints

The inequality constraints that are being handled dictate the required approach. Two approaches are employed in this work to handle the inequality constraints. These approaches are summarized below;

1. The constraints associated with voltage magnitudes and the real power generations are usually made inactive in the initial solution process [24]. But when it is violated, they are handled by activating the penalty function of (9) [37, 38]. After the activation, the first and second partial derivatives corresponding to the affected inequality constraint are added to the elements of the gradient vector and the Hessian matrix respectively. The linearized Equation (14) is again solved and the variables updated using (15).

2. Since reactive power at a generator bus is a function of some systems variables, it becomes a functional inequality constraint, hence, handling such constraints requires the usage of a form of penalty function given in (10) [37,38]. If the reactive power at a generator bus is within limit, the first and second partial derivatives of (10) are, respectively, added to the elements associated to $\lambda_{qi}(t)$ in the Hessian matrix and gradient vector of (14) to deactivate the reactive power flow equation at generator bus i. However, these partial derivatives are removed when reactive inequality constraint becomes activated.

It is important to know that multipliers μ and weighing factors c and *S* are updated according to the criteria given in [37].

4.3 Adjustment Strategy for Water worth Variable

It can be observed that the solution to (14) does not cater for the water worth variable (υ). This is to reduce the size of the gradient vector and Hessian matrix to be handled, as smaller size of gradient vector and Hessian matrix guarantees better and reliable solution than the ones with larger size.

In this work, the strategy adopted to adjust the water worth variable requires the element of the gradient vector that relates to the active power output of the hydro station ($P_h(t)$). At bus *i* with hydro station *h* and during time interval *t*, it is definite that the expression for this element is given as:

$$\frac{\partial L_t(z,\lambda,\upsilon)}{\partial P_h(t)} = \upsilon_h\left(\beta_h + 2\gamma_h P_h(t)\right) - \lambda_{pi}(t) = 0 \quad (16)$$

From (16), $P_h(t)$ is derived as:

$$P_h(t) = \frac{\lambda_{pi}(t) - \beta_h \upsilon_h}{2\gamma_h \upsilon_h} \quad (17)$$

Substituting for $P_h(t)$ in (5) yields:

$$q_h = \sum_{i=1}^{T}\left(\alpha_h + \beta_h\left(\frac{\lambda_{pi}(t)}{2\gamma_h v_h} - \frac{\beta_h}{2\gamma_h}\right) + \gamma_h\left(\frac{\lambda_{pi}(t)}{2\gamma_h v_h} - \frac{\beta_h}{2\gamma_h}\right)^2\right) \quad (18)$$

So,

$$f(\upsilon_h) = q_h \quad (19)$$

Expanding the left-hand side of (19) in Taylor's series about an operating point $\upsilon_h^{(m)}$, and neglecting the higher-order terms results in:

$$f(\upsilon_h)^{(m)} + \left(\frac{df(\upsilon_h)}{d\upsilon_h}\right)^{(m)} \Delta\upsilon_h^{(m)} = q_h \quad (20)$$

$$\Delta\upsilon_h^{(m)} = \frac{q_h - f(\upsilon_h)^{(m)}}{\left(\frac{df(\upsilon_h)}{d\upsilon_h}\right)^{(m)}} = \frac{\Delta q_h^{(m)}}{\left(\frac{df(\upsilon_h)}{d\upsilon_h}\right)^{(m)}} \quad (21)$$

Simplifying (21) gives:

$$\Delta\upsilon_h^{(m)} = -\frac{\Delta q_h^{(m)}}{\left(\frac{1}{2\gamma_h v_h^3}\sum_{i=1}^{T}\lambda_{pi}(t)^2\right)^{(m)}} \quad (22)$$

Therefore, at iteration *m + 1*, the water worth value is adjusted using the equation below:

$$\upsilon_h^{(m+1)} = \upsilon_h^{(m)} + \Delta\upsilon_h^{(m)} \quad (23)$$

This adjustment is repeated after every solution to the Lagrangian function for all the time intervals until the available water is optimally utilized.

4.4 Flow Chart for the Decomposition Approach

The flow chart for the proposed HTOPF procedure is given in Fig. 1. Five main steps are identified in the flow chart: (1) *variable initialization* (2) *iteration loop 1* (3) *iteration loop 2* (4) *optimization interval loop 3* and (5) *iteration loop 4*. The iteration loops are represented by the arrow head arc with loop number written inside it.

The first step is proper *variable initialization* for a good rate of convergence. In this work, voltage magnitude at all buses is initialized at 1.0 per unit. The voltage angles at all buses are initialized at 0^o. These values are similar to that of the power flow solution procedure. The Lagrange multipliers for reactive power flow mismatch equation is initialized at zero. To initialize the bus Lagrange multiplier relating to active power flow mismatch equation, water worth value and active power output of both the hydro and thermal resources for every time interval, a simple hydro-thermal dispatch is adopted [37]. It is important to note that, this dispatch assumes that the loads and generation are connected to a bus without the consideration of system configuration, line impedances, losses and limits [39].

Iteration loop 1 solves the linearized Equation (14) and updates the HTOPF variables except the water worth value. This loop also makes sure that the gradient vector mismatch is less than the tolerable limit of 10^{-4}. *Iteration loop 2* ensures that all the HTOPF variables from loop 1 are within bound. If iteration loop 2 is activated due to variable limits violation, the elements of the gradient vector and Hessian matrix relating to the violated limit are, respectively, adjusted. Since gradient vector and Hessian matrix are adjusted, iteration loop 1 is definitely needed to ensure that the adjusted gradient vector mismatch is again less than the tolerable limit of 10^{-4}. This continues until variables are within bound. After the enforcement of limit on all variables that are outside bound, *optimization interval loop 3* is needed to allow for solving the next optimization interval *t*. At a given interval *t=T*, *iteration loop* 4 is entered to check if water availability mismatch is less than the tolerable limit of 10^{-4} and if it is not, an update of the water worth value is required. This procedure continues until water energy constraint is satisfied.

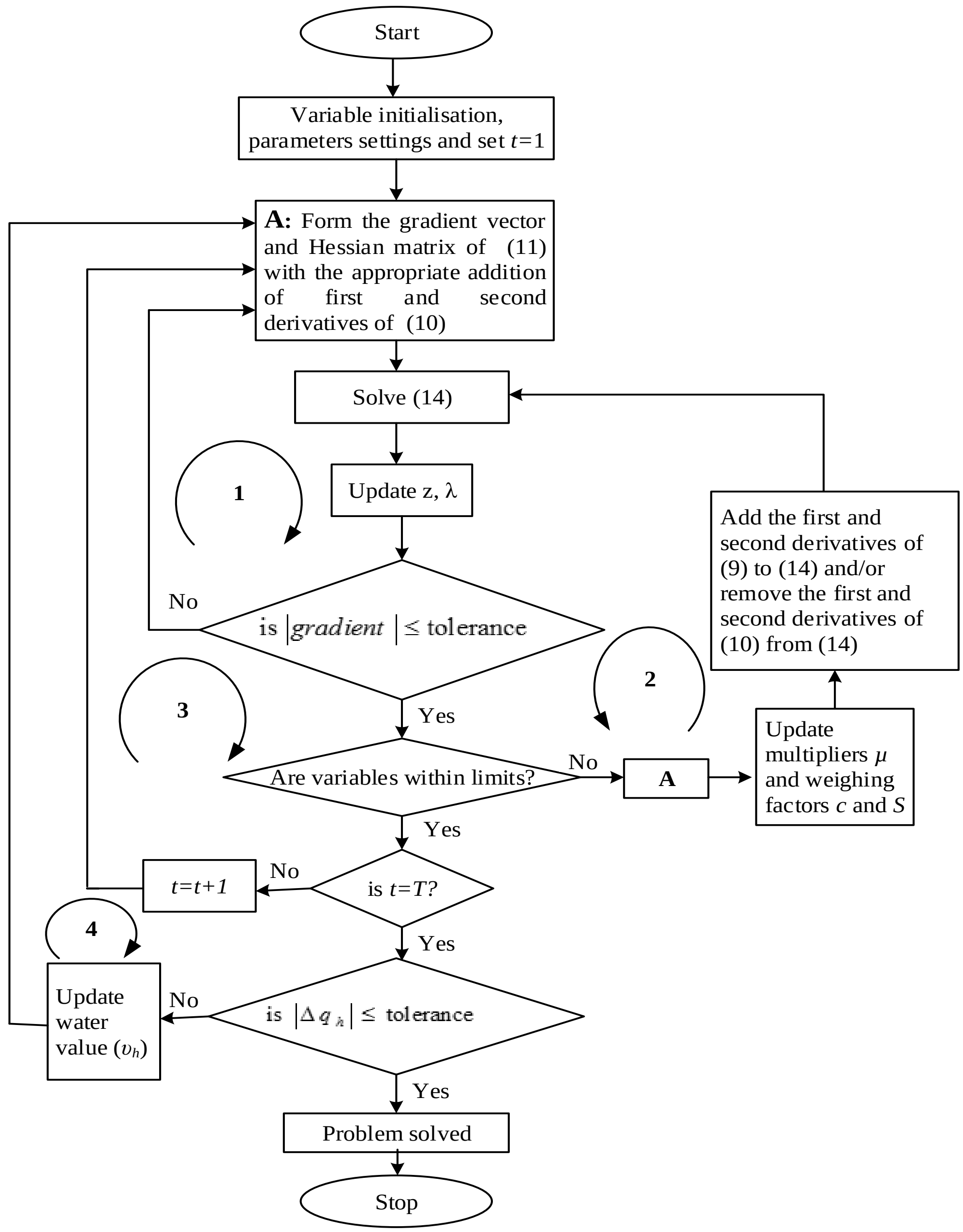

Fig. 1. Flow chart for the decomposed Newton-Based hydro-thermal optimal power flow solution technique

This flow chart has been implemented in MATLAB software environment.

5. RESULTS AND DISCUSSION

The proposed algorithm has been implemented using MATLAB version 7.14 software. The results of the validation and evaluation of the algorithm is presented in this section. The validation has been done using the developed program to solve some similar HTOPF problems presented in the existing work [3]. In order to evaluate the developed software, the developed program has been used to further solve the HTOPF problem of some power systems; the IEEE 5-bus, 30-bus and the 57-bus networks. A Twenty-four hour load duration interval is considered for this work. The load curve used for this study is shown in Fig. 2 [30]. The iteration referred to in this section is the iteration loop 4 discussed in Section 4.4.

5.1 Assessment of the Decomposition Approach

To validate the performance of the developed program, it has been used to solve the HTOPF problem of 5-bus and 30-bus systems using the load curves (i.e. three time intervals for 5-bus and two time intervals for other systems) presented in [3]. The hydro and thermal generators' characteristics presented in [3] have also been used for this assessment.

The results obtained are compared with the results presented in [3]. The results for 5-bus and 30-bus systems are, respectively, shown in Tables 1 and 2. From the tables, it is observed that the results compared well with those of existing Newton-based technique with slight differences presented. For instance, the cost of fuel obtained for the proposed approach is slightly less than the one presented in [3] for 5-bus. While the cost of generation obtained for the proposed approach is insignificantly higher when compared to the one presented in [3] for the 30-bus system. However, the proposed HTOPF solution procedure has the ability to accommodate more intervals with varying system's constraints and loading conditions which is a common feature of the deregulated power market.

5.2 Description of Test Systems

The data for 5-bus, 30-bus and 57-bus are sourced from past works [3, 37, 39, 40]. The characteristics of these systems are summarized in Table 3. Where *ntl* is the number of transmission lines, *nld* is the number of load buses, *nt* and *nh* are as earlier defined. The cost and discharge characteristics of the thermal and hydro generators of the test systems are contained in [41].

5.3 HTOPF Results

The optimal schedules of the hydro and thermal generators are presented in this section. The parameters that describe the optimal schedules are; the total energy generated, the total system transmission loss, the total cost of fuel for thermal plant generation, the total cost of water for the hydro plant, maximum and minimum voltage magnitude of the system, water worth and average energy of the hydro plants. The total cost of water (i.e. savings in fuel cost) is derived

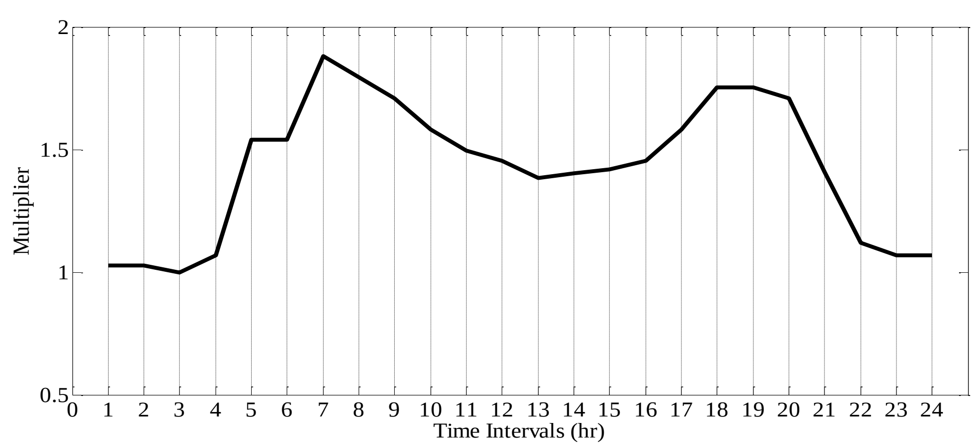

Fig. 2. Load curve for the test systems [30]

Table 1. Comparison of results for 5-bus

interval		1	2	3	Total
thermal genration at bus 1 (MW)	existing approach [3]	48.52	72.28	64.34	185.1
	proposed approach	47.63	72.89	64.43	184.9
hydro generation at bus 2 (MW)	existing approach [3]	68.56	95.99	86.79	251.3
	proposed approach	69.38	95.34	86.65	251.4
Cost ($)	existing approach [3]	588.38	870.24	774.66	2233.3
	proposed approach	578.08	877.65	775.75	2231.5

Table 2. Comparison of results for 30-bus

interval		1	2	Total
thermal generation at bus 1 (MW)	existing approach [3]	50.88	97.58	148.5
	proposed approach	48.47	93.83	142.3
thermal generation at bus 13 (MW)	existing approach [3]	52.52	101.9	154.4
	proposed approach	54.11	106.4	160.5
hydro generation at bus 2 (MW)	existing approach [3]	41.05	93.63	134.7
	proposed approach	41.47	93.24	134.7
Cost ($)	existing approach [3]	1271.96	2466.11	3738.07
	proposed approach	1262.46	2476.55	3739.01

Table 3. Characteristics of the test systems

no of buses	ntl	nt	thermal generator(s) location	nh	hydro generator(s) location
5	7	1	bus 1	1	bus 2
30	41	4	buses 1, 2, 8, 11	2	buses 5, 13
57	80	5	buses 1, 2, 3, 6, 9	2	Buses 8, 12

from the last term of Equation (8). The average hydro plant energy (AHPE) for hydro plant *h* is calculated using (24). It should be noted that the active power by thermal generator at bus *J* and hydro generator at bus *K* are, respectively, represented as *PgtJ* and *PhtK*. For example, thermal generator at bus 1 is represented as *Pgt1* and hydro generator at bus 2 is represented as *Pht2*.

$$\text{AHPE} = \frac{\sum_{t=1}^{T} P_{ht}}{q_h} \tag{24}$$

5.3.1 Results for 5-bus System

The HTOPF solution has been obtained in fifteen iterations with an absolute maximum water availability mismatch of 6.5×10^{-06}. The maximum and minimum voltage magnitudes during the optimization intervals are, respectively, 1.0413 pu and 0.9724 pu. The optimal water worth value and AHPE are 83.69 $\$/Mm^3$ and 3.9546 MWH/Mm^3, respectively. The total cost of water for the hydro plant is $58,618.79. The total energy generated is 5,833.76 MWH and 47.48% of this energy is from the hydro station. The contribution of hydro generator depends on the discharge characteristics and water availability of the hydro plant. The total fuel cost for generation is $68,678.30 while the total transmission loss is 185.61 MWH and this value amounts to 3.18% of the total energy generation. The optimal schedules of the two generators are shown in Fig. 3. It can be seen that *Pgt1* is constant at 90 MW at hours 1 to 4 and 22 to 24. This is because of the minimum mega-watt limit of the unit. It can also be seen that thermal unit at bus 1 generate more power almost throughout the optimization period.

5.3.2 Results for 30-bus System

The HTOPF solution of this system has been achieved with absolute maximum water mismatch of 8.18×10^{-06} after eighteen iterations. The maximum and minimum voltage magnitudes are, respectively, 1.0994 pu and 0.9007 pu. The water worth values for hydro stations at buses 5 (HS5) and 13 (HS13) are 28.93 $\$/Mm^3$ and 10.91 $\$/Mm^3$, respectively. The higher water worth value of HS5 shows that, it better reduces the total cost of generation when compared to HS13. The total energy generation amounts to 9,824.4 MWH. The contribution of the hydro station to the total generation is 47.97 % with a total cost of water of

\$12,019.86. The AHPE for HS5 and HS13 are, respectively, 11.6813MWH/Mm^3 and 4.009 MWH/Mm^3. The average water energy indicates that HS5 better utilizes its water than HS13. The total fuel cost for the entire period is \$11,700.98 while the total transmission loss is 123.32 MWH and this is 1.26% of the total energy generation. The active schedules of the generation stations in the system are shown in Fig. 4. This schedule has shown that the hydro unit at bus 5 generates more power throughout while the thermal unit at bus 11 generates the lowest amount of power.

5.3.3 Results for 57-bus System

The HTOPF solution converges in nine iterations to a water worth value of 394.96 \$/Mm^3 and 188.67 \$/Mm^3 for HG8 and HG12 respectively. The contribution of the hydro stations to the total generation is 31.40 % while the AHPE from HG8 and HG12 are respectively 10.8826 MWH/Mm^3 and 4.0164 MWH/Mm^3. The AHPE for both hydro generators show that HG8 maximises water better than HG12. The maximum and minimum voltage magnitudes are, respectively, 1.099 and 0.9003 per unit. The absolute maximum water availability mismatch is 5.90 $\times 10^{-6}$. The total cost of generation for the entire optimization interval is \$617,588.75. The optimal schedules of both thermal and hydro generators are shown in Fig. 5. It is clear from this figure that the 250 MW limit on *Pgt6* has caused the almost constant power outputs at hours 7 and 8. It is also obvious that the hydro unit at bus 8 generates more power during the period under consideration.

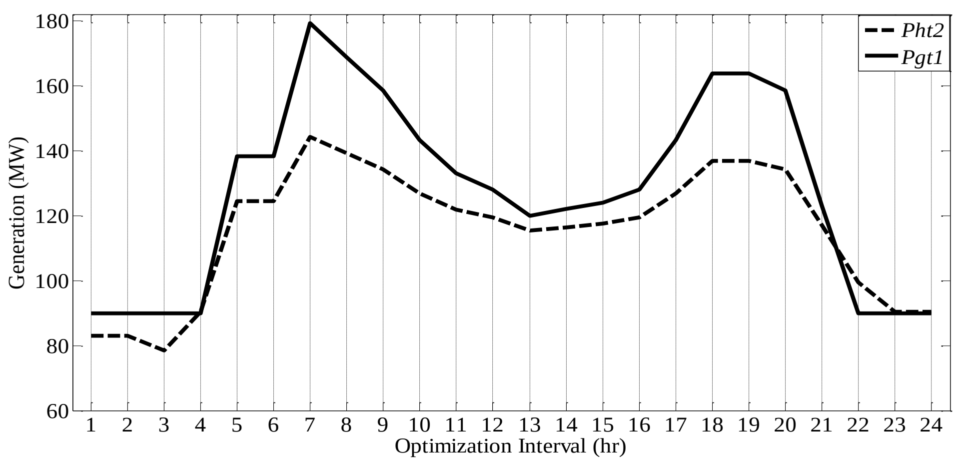

Fig. 3. HTOPF active schedule of generators for 5-bus system

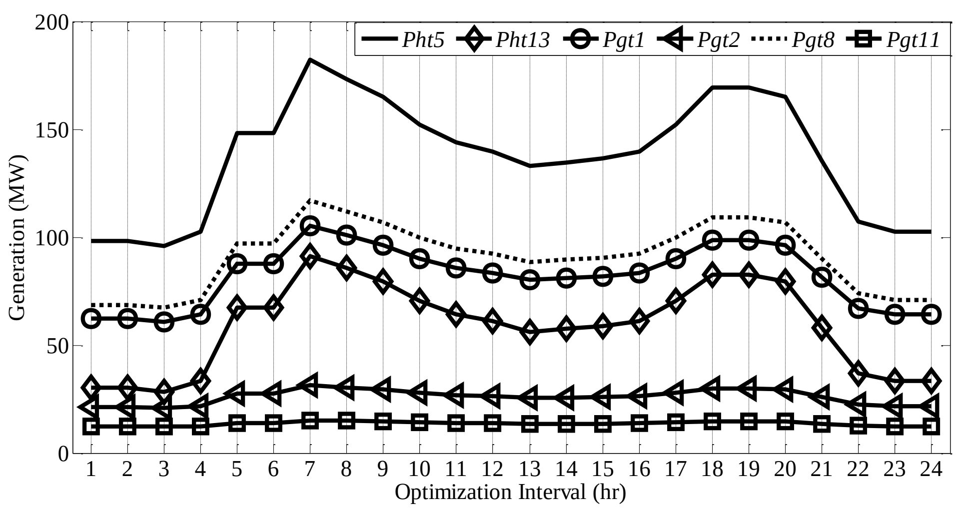

Fig. 4. HTOPF active schedule of generators for 30-bus system

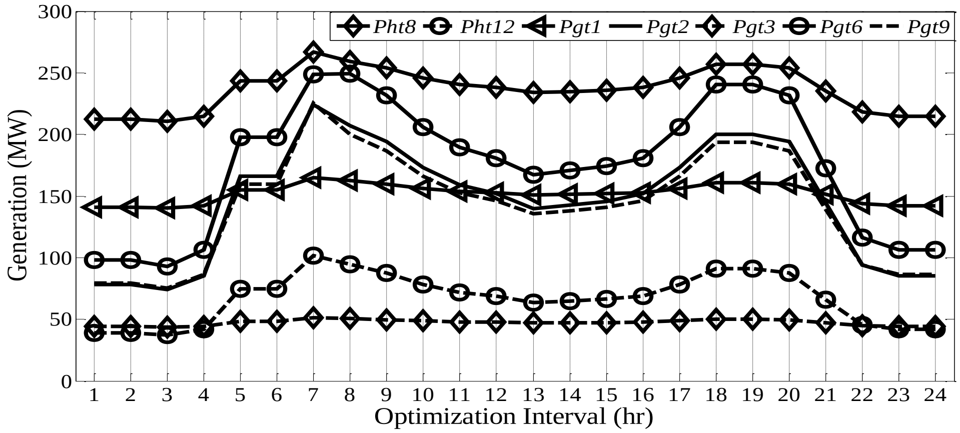

Fig. 5. HTOPF active schedules of generators for 57-bus system

6. CONCLUSION

A decomposition approach to the Newton-based method of solving the HTOPF problem is carried out in this work. The proposed approach caters for the gap created due to increased system size and (or) time interval. The approach is achieved by first handling each interval separately for the period considered and updating the water worth value until the water availability constraint is satisfied. The HTOPF is formulated as an optimization problem with cost of generating active power as the objective function and power balance equations as the equality constraints. Some system's inequality constraints are also considered. MATLAB software program has been developed to implement the solution procedure. The developed software has been tested on standard power systems to evaluate its performance and the results obtained are reported. Since the HTOPF problem has been decomposed into hourly subproblem, the computational efficiency is guaranteed.

The comparisons of the results of the existing and the proposed HTOPF Newton's solution procedures show that both approaches are comparable for the time intervals considered (i.e. three intervals for 5-bus system and two intervals for 14-bus and 30-bus systems). However, the proposed algorithm is capable of handling more intervals with varying loading conditions. As a result of this property, the proposed methodology is suitable for Independent System Operator.

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