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LINEAR QUADRATIC GAUSSIAN (LQG) CONTROL DESIGN FOR THE BALL-ON-SPHERE SYSTEM

Ezechukwu Kalu Ukiwe[1], Suleiman Usman Hussein[2], Omotayo Oshiga[3]
[1,2&3]Department of Elect. & Elect. Engineering, *Nile University,* Abuja – Nigeria.
[1]ezeukiwe@yahoo.com, [2]elsuligh@gmail.com, [3]ooshiga@nileuniversity.edu.ng
Corresponding author email: ezeukiwe@yahoo.com

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ABSTRACT

This paper is geared towards controlling the Ball on Sphere (BOS) system by adopting the model developed in similar research for the ball on ball system and modified for control of the ball on sphere system however implemented using Linear Quadratic Gaussian (LQG) Control scheme. The linearized model for controlling a BOS system was adopted and configured. These configurations were then used to compute the state and input matrices taking consideration of the covariance Gaussian white noise elements and other parameters like the Kalman filter to create the Linear Quadratic Gaussian (LQG) controller. The result was simulated with MATLAB 2019b application which indicates that the BOS system is controllable, observable and stable. The paper assumes that the ball is always in direct contact with the sphere and that there is no translational motion between the ball and the sphere.

1. INTRODUCTION

The LQG is basically a loop within a loop control system that uses the double feedback loop structure. Linear Quadratic Gaussian Control of the ball on plate [1], all geared to stabilizing a ball on top of another element. Such control problem reveals the overarching importance of balancing systems in modern control. The concept of balancing systems therefore continues to elicit ever increasing research in virtually all aspects of control systems, due in part to their varied applications in astronomy, navigation, tracking, etc.

The successful modelling of the single Ball on Sphere system provides the platform for attempting more challenging cases of multiple balls on sphere systems.

In another sense, the BOS is a rather complicated system compared with any of the Ball on Beam (BOB), Ball on Wheel (BOW), and Ball on Plate (BOP) systems. Though similar to the Ball on Ball system which is a special case of the BOS wherein the physical characteristics of the both balls are the same. This poses challenge not only in terms of controllability and stability as much as they are non-linear systems [2],[3], but also in system performance by virtue of chosen controller mechanism. Notwithstanding the popularity garnered by Proportional Integral Derivative (PID) based controllers, their associated inherent drawbacks [3] necessitated the choice of LQG for this paper.

The BOS in this paper is analysed with a twin feedback rings structure for orientation, motion, and control. The outer feedback loop is developed using linear quadratic gaussian controller, while the inner loop accomplished with requisite linear quadratic regulator equations.

The next section of this paper highlights the adopted mathematical models of the BOS, while the succeeding sections reviews the design of the controllers and the simulated results of system with conclusion in unit 6.

2. LITERATURE REVIEW

Works on control of similar non-linear systems [2] had been done in the past for instance in Fuzzy logic control of a ball on sphere system by [4] wherein the authors contends that fuzzy Logic control framework can mimic rational analysis and “linguistic control ability” in order to “equip the control system with certain degree of artificial intelligence. They highlighted the use of Adaptive Neural Network for controlling a Multi-Input, Multi-Output (MIMO) non-linear system like the BOS system when subjected to random disturbances. They showed the inherent accuracy and imperative of applying such controller mechanism. Hence, applied adaptive algorithm to develop the controller and a type of open loop neural network technique called radial basis network generated from the radial basis activation function. Though the method creates quite definite result, the complexity involved in applying gaussian radial basis function could be quite involving unlike the LQG that is simpler to implement.

In another instance, [1], while using the LQG technique to stabilize the BOP found that the method exhibited good performance characteristics. The author used of the H-infinity sensitivity method to design the outer feedback loop. According to [5], the H-infinity controller can easily meet the requisite high level step response performance by merely selecting the weighting functions as appropriate, however it is somewhat constrained by sub-optimal input stability margins. And, given that the BOP have 4 DOF, this paper ventures to validate the possibility of applying the LQG on more complex systems with higher order DOF like the BOS system.

Whereas [6] developed a model for the Ball on Ball System on the basis that the non-holonomic constraints posed by rotating a spherical object on top of another identical spherical one controlled by three (3) wheels so that the system can rotate in any direction (x,y,z) coordinates. However, his work focused excellently on deriving a baseline model that can be adopted to control any ball on ball system. Little attention was given to the analyzing the controller system responses.

In trying to restate the attractiveness of non-linear control theory [2], applied adaptive feedback linearization method to regulate the BOS system. The authors concluded that such linearization mechanism can offer asymptotically accurate compensation for randomness inherent in the system characteristics. Notwithstanding the differences in controller design, another area of divergence between their work and this one lies in the focus of their study on the system output characteristics whereas this paper concentrated analysis on the individual step responses of system variables. In their work, [7] investigated the effect of unwanted noise elements in their random states in the control of non-linear systems and opined that it is better to use the kalman filter to estimate the state variables than the feedback linearization method due to the later susceptibility to noise. Again, their analysis was restricted to the output variables.

While studying the impact of friction on the BOS system, [8] applied the bond graph modelling methodology and analyzed the system angular displacement dynamic responses with superlative results. However, their investigation was narrowed to only angular positions in the x-axis without observing same parameter in the y- or z-axes; and overlooked the other parameters like system angular velocities in all the axes including the x –axis, which was the focus of their analysis

Though [2] in their model did not consider completely the mass of the sphere but its straightforward nature makes it quite easy to understand and redevelop for more challenging models. The model by [2] was therefore used in this paper but with parametric variations of the physical characteristics of the ball and underlying sphere. More areas of research exist especially in analyzing the BOS system responses in the frequency domain.

3. SIMULATION ENVIRONMENT

Model of the Ball on Sphere System in 3D

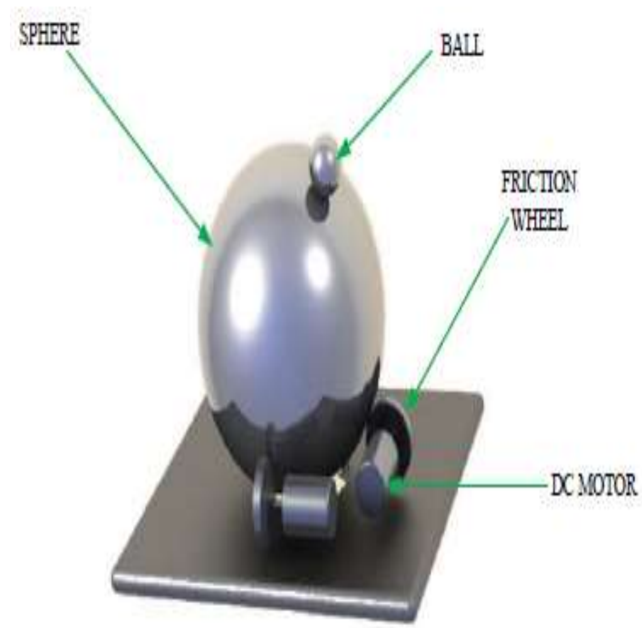

Figure 1: Simple Model of the Ball on Sphere System (BOS) [2],[8]

Figures 1 shows simple 3D model of the BOS system whereas Figure 2 indicates the schematic model of the BOS system in the cartesian coordinate system.

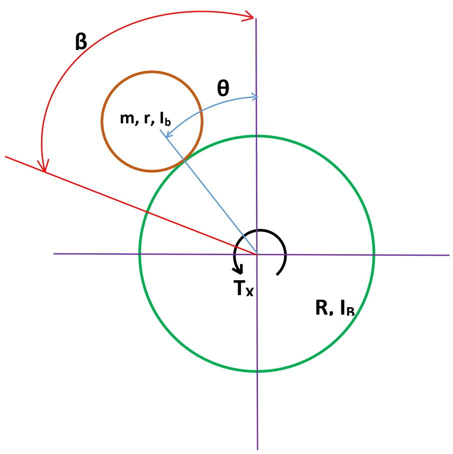

Figure 2: Model of the BOS in the Cartesian Coordinate System[1], [4]

The model uses the second order Lagrange equations wherein the power train produces the necessary torques on the sphere through omni-wheels, whereas the task of the controller is to balance the ball on the sphere in the event of rotary or semi-rotary motion of the sphere caused by inputs from the actuating element (drive train) [1].

The ball with the radius (r) is treated as a homogenous rigid body with the mass m, the same as the sphere with corresponding mass M. Their respective moment of inertia I and J are modelled as a diagonal matrix

Analysis of the Spatial rotation of the ball is accomplished by means of the Euler angles with respect to the z − x − z convention for the rotation of a reference body so that the angular velocity vector of the reference frame in accordance with [1] the Lagrange equations for a system with n generalized coordinates and nonholonomic constraints [9], [10], [11] have the form:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q_i, \qquad i = 1,2,3,4$$

With the Langrangian L = T-V (Where T and V are the Kinetic and Potential energies of the system). Where q is the generalized coordinate and i is an integer.

$$Q_1 = 0 \tag{3}$$

$$Q_2 = T_x \tag{4}$$

$$Q_3 = 0 \tag{5}$$

$$Q_4 = T_y \tag{6}$$

$$\left((R+r)m + I_b\frac{R+r}{r^2}\right)\ddot{\theta}_x + \left(-I_b\frac{R}{r^2}\right)\ddot{\beta}_x - mg\sin(\theta_x) = 0 \tag{7}$$

$$\left(-I_b\frac{R(R+r)}{r^2}\right)\ddot{\theta}_x + \left(I_B + I_b\frac{R^2}{r^2}\right)\ddot{\beta}_x = T_x \tag{8}$$

$$\left((R+r)m + I_b\frac{R+r}{r^2}\right)\ddot{\theta}_y + \left(-I_b\frac{R}{r^2}\right)\ddot{\beta}_y - mg\sin(\theta_y) = 0 \tag{9}$$

$$\left(-I_b\frac{R(R+r)}{r^2}\right)\ddot{\theta}_y + \left(I_B + I_b\frac{R^2}{r^2}\right)\ddot{\beta}_y = T_y \tag{10}$$

$$q = [\theta_x \quad \beta_x \quad \theta_y \quad \beta_y] \tag{11}$$

$$M\ddot{q} + G = T \tag{12}$$

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \tag{13}$$

$$M_{11} = (R + r)m + I_b\frac{R+r}{r^2}$$

$$M_{12} = -I_b\frac{R}{r^2}$$

$$M_{13} = 0$$

$$M_{14} = 0$$

$$M_{21} = -I_b\frac{R(R+r)}{r^2}$$

$$M_{22} = I_B + I_b\frac{R^2}{r^2}$$

$$M_{23} = 0$$

$$M_{24} = 0$$

$$M_{31} = 0$$

$$M_{32} = 0$$

$$M_{33} = (R + r)m + I_b \frac{R+r}{r^2}$$

$$M_{34} = -I_b \frac{R}{r^2}$$

$$M_{41} = 0$$

$$M_{42} = 0$$

$$M_{43} = -I_b \frac{R(R+r)}{r^2}$$

$$M_{44} = I_B + I_b \frac{R^2}{r^2}$$

$$G = \begin{bmatrix} -mg\sin(q_1) \\ 0 \\ -mg\sin(q_3) \\ 0 \end{bmatrix} \quad (14)$$

$$T = \begin{bmatrix} 0 \\ T_x \\ 0 \\ T_y \end{bmatrix} \quad (15)$$

So

$$\ddot{q} = M^{-1}(T - G) \quad (16)$$

State space variables:

$$x_1 = \theta_x \quad (17)$$

$$x_2 = \dot{\theta}_x \quad (18)$$

$$x_3 = \beta_x \quad (19)$$

$$x_4 = \dot{\beta}_x \quad (20)$$

$$x_5 = \theta_y \quad (21)$$

$$x_6 = \dot{\theta}_y \quad (22)$$

$$x_7 = \beta_y \quad (23)$$

$$x_8 = \dot{\beta}_y \quad (24)$$

$$\dot{x}_1 = x_2 \quad (25)$$

$$\dot{x}_2 = \ddot{q}_1 \quad (26)$$

$$\dot{x}_3 = x_4 \quad (27)$$

$$\dot{x}_4 = \ddot{q}_2 \quad (28)$$

$$\dot{x}_5 = x_6 \quad (29)$$

$$\dot{x}_6 = \ddot{q}_3 \quad (30)$$

$$\dot{x}_7 = x_8 \quad (31)$$

$$\dot{x}_8 = \ddot{q}_4 \quad (32)$$

$$a_{01} = (R + r)m \quad (33)$$

$$a_{02} = I_b \left(\frac{R}{r^2}\right) \quad (34)$$

$$a_{03} = I_b \left(\frac{R^2}{r^2}\right) \quad (35)$$

$$a_{04} = I_B \quad (36)$$

$$a_{05} = \frac{I_b}{r} \quad (37)$$

$$a_{06} = I_b \frac{R}{r} \quad (38)$$

$$a_{07} = m \quad (39)$$

$$\rho = \begin{bmatrix} a_{01} \\ a_{02} \\ a_{03} \\ a_{04} \\ a_{05} \\ a_{06} \\ a_{07} \end{bmatrix} \quad (40)$$

As a result;

$$M = \begin{bmatrix} a_{01}+a_{02}+a_{05} & -a_{02} & 0 & 0 \\ -a_{03}-a_{06} & a_{04}+a_{03} & 0 & 0 \\ 0 & 0 & a_{01}+a_{02}+a_{05} & -a_{02} \\ 0 & 0 & -a_{03}-a_{06} & a_{04}+a_{03} \end{bmatrix} \quad (41)$$

$$G = \begin{bmatrix} -a_{07} g \sin(\theta_x) \\ 0 \\ -a_{07} g \sin(\theta_y) \\ 0 \end{bmatrix} \quad (42)$$

$$W = \begin{bmatrix} \ddot{\theta}_x & \ddot{\theta}_x - \ddot{\beta}_x & 0 & 0 & \ddot{\theta}_x & 0 & 0 \\ 0 & 0 & \ddot{\beta}_x - \ddot{\theta}_x & \ddot{\beta}_x & 0 & -\ddot{\theta}_x & -g\sin(\theta_x) \\ \ddot{\theta}_y & \ddot{\theta}_y - \ddot{\beta}_y & 0 & 0 & \ddot{\theta}_y & 0 & 0 \\ 0 & 0 & \ddot{\beta}_y - \ddot{\theta}_y & \ddot{\beta}_y & 0 & -\ddot{\theta}_y & -g\sin(\theta_y) \end{bmatrix} \quad (43)$$

As a result:

$$M\ddot{q} + G = W\rho \quad (44)$$

Assuming T_x and T_y then ,θ_x, β_x, θ_y and β_y to be, respectively, the system's inputs and outputs. By random numbering of the state variables, we have;

$$x_1 = \theta_x \quad (45)$$

$$x_2 = \beta_x \quad (46)$$

$$x_3 = \theta_y \quad (47)$$

$$x_4 = \beta_y \quad (48)$$

$$x_5 = \dot{\theta}_x \quad (49)$$

$$x_6 = \dot{\beta}_x \quad (50)$$

$$x_7 = \dot{\theta}_y \quad (51)$$

$$x_8 = \dot{\beta}_y \quad (52)$$

We get, from (45) – (52), the first 4 state equations as;

$$\dot{x}_1 = x_5 \quad (53)$$

$$\dot{x}_2 = x_6 \quad (54)$$

$$\dot{x}_3 = x_7 \quad (55)$$

$$\dot{x}_4 = x_8 \quad (56)$$

In order to improve the algebraic linearity of the system [12], the equations are further linearized about the equilibrium position by assuming the angles of the ball (θ) and that of the sphere (β), to be negligible [13], [14]. Such that, $\sin(\theta) = \theta$, $\cos(\theta) = 1$, $\sin(\beta) = \beta$, and $\cos(\beta) = 1$. We apply these assumptions into equations (7) – (10) to get;

$$a\ddot{\theta}_x + b\ddot{\beta}_x - mg\theta_x = 0 \quad (57)$$

$$c\ddot{\theta}_x + d\ddot{\beta}_x = T_x \quad (58)$$

$$a\ddot{\theta}_y + b\ddot{\beta}_y - mg\theta_y = 0 \quad (59)$$

$$c\ddot{\theta}_y + b\ddot{\beta}_y = T_y \qquad (60)$$

For:

$a = \left((R + r)m + I_b \frac{R+r}{r^2}\right), b = \left(-I_b \frac{R}{r^2}\right),$

$c = \left(-I_b \frac{R(R+r)}{r^2}\right)$, and $d = \left(I_B + I_b \frac{R^2}{r^2}\right)$

from where we get the other 4 state equations as follows;

$$\dot{x}_5 = vx_1 - wT_x \qquad (61)$$

$$\dot{x}_6 = zx_1 - eT_x \qquad (62)$$

$$\dot{x}_7 = vx_3 - wT_y \qquad (63)$$

$$\dot{x}_8 = zx_3 - eT_y \qquad (64)$$

For:

$v = \frac{dmg}{ad-bc}$, $w = \frac{b}{ad-bc}$, $z = \frac{cmg}{bc-ad}$, and $e = \frac{a}{bc-ad}$,

Which can then be expressed in matrix form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -w & 0 \\ -e & 0 \\ 0 & -w \\ 0 & -e \end{bmatrix} \begin{bmatrix} T_x \\ T_y \end{bmatrix} \qquad (65)$$

The output equations are;

$$\theta_x = x_1 \qquad (66)$$

$$\beta_x = x_2 \qquad (67)$$

$$\theta_y = x_3 \qquad (68)$$

$$\beta_y = x_4 \qquad (69)$$

In matrix form,

$$\begin{bmatrix} \theta_x \\ \beta_x \\ \theta_y \\ \beta_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_x \\ T_y \end{bmatrix} \qquad (70)$$

And the coefficient matrices are;

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -w & 0 \\ -e & 0 \\ 0 & -w \\ 0 & -e \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (71)$$

4 Design

Linear Quadratic Gaussian (LQG) Controller

Linear Quadratic Gaussian (LQG) controller is widely accepted robust control method in the sense that it offers more realistic dynamical model which can be utilized for the development of the control scheme by considering both plant disturbances and measurement / sensor noises at the same time [15].

The dynamic model applied for the controller synthesis is the random dynamic model allowing consideration of both plant disturbances and sensor noises inherent in the LQG control system. This is because LQG controllers have in-built or modelled with considerations of controller noise and measurement noise due to the metering / sensor devices.

The LQG compensator can be represented in a block diagram as given in Figure 3 below.

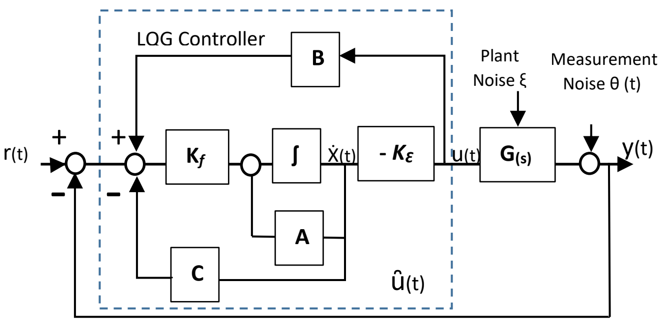

Figure 3: Control Structure of the LQG [5]

The optimal control $u^*(t)$ is given as:

$$u^*(t) = -K_c \hat{x}(t) \tag{72}$$

Also, the optimal state feedback matrix K_c is written as:

$$K_c = R^{-1} B^T P_c \tag{73}$$

And the symmetrical semi-positive-definite matrix which confirms the Algebraic Riccatti equation, is represented by equation (74) [16]:

$$A^T P_c + P_c A - P_c B R^{-1} B^T P_c + M^T Q M = 0 \tag{74}$$

Based on the separation principle, the solution of the optimal control problem can be resolved independent of the optimal estimation one [1], [15]. Wherein for the LQG controller, the state estimator is developed and used to derive the Linear Quadratic Regulator (LQR) state feedback controller design on the assumption that the states are precisely measurable for the LQR state feedback controller scheme.

First, the Kalman-Estimator optimal static gain which helps to reconstitute the estimated state vector (using Linear Quadratic Estimator (LQE) problem) is derived; then the optimal static feedback gain matrix found in solution of the LQR (Linear Quadratic Regulator) is computed [15].

The system physical characteristics are as follows:

R	–	0.25m
r	-	0.0125m
m	-	0.06kg
M	–	0.5kg
I	-	3.75 x 10^{-6}kgm^2
J	-	0.0125kgm^2
g	-	9.8m/s^2

5 Results and Discussion

Table 1: System & Initial Parameters

Parameters	**Rise Time (s)**	**Settling Time (s)**	**Peak Time (s)**
Ball Ang. Disp. in X axis **(θ_x)**	0.7474	12.8025	8
Sphere Ang. Disp. in X axis **(β_x)**	0.7474	12.8244	8
Ball Ang. Disp. in Y axis **(θ_y)**	0.7474	12.8420	8
Sphere Ang. Disp. in Y axis **(β_y)**	0.7474	12.8564	8
Ball Ang. Velocity in X axis **(θ'_x)**	0.7474	12.8683	8
Sphere Ang.Velocity in X axis **(β'_x)**	0.7474	12.8785	8
Ball Ang. Velocity in Y axis **(θ'_y)**	0.7474	12.8871	8
Sphere Ang.Velocity in Y axis **(β'_y)**	0.7474	12.8947	8

The design was simulated with MATLAB 2019b software and the following results were summarized in Table 1.

Step responses of LQG controller are shown Figures 4

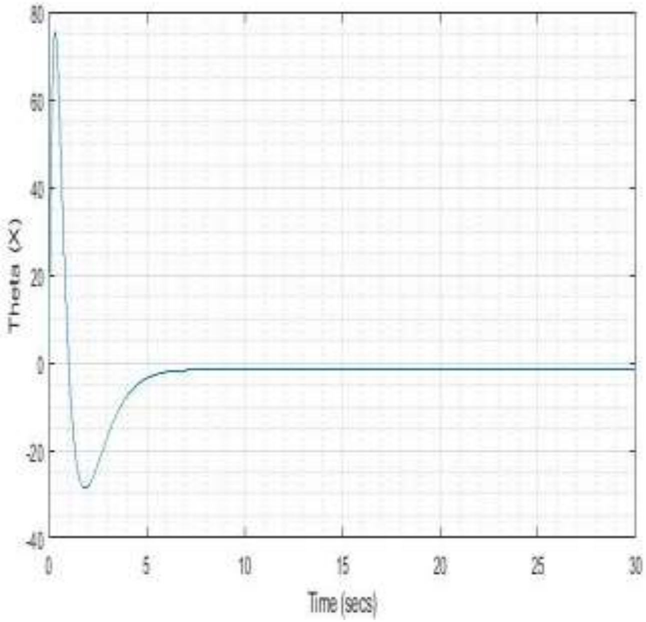

Fig 4a: Θ_x - Ball Angular Displacement in X-axis

Fig 4a above indicates how the system was able to stabilize in 12.8025 seconds when subjected to disturbance.

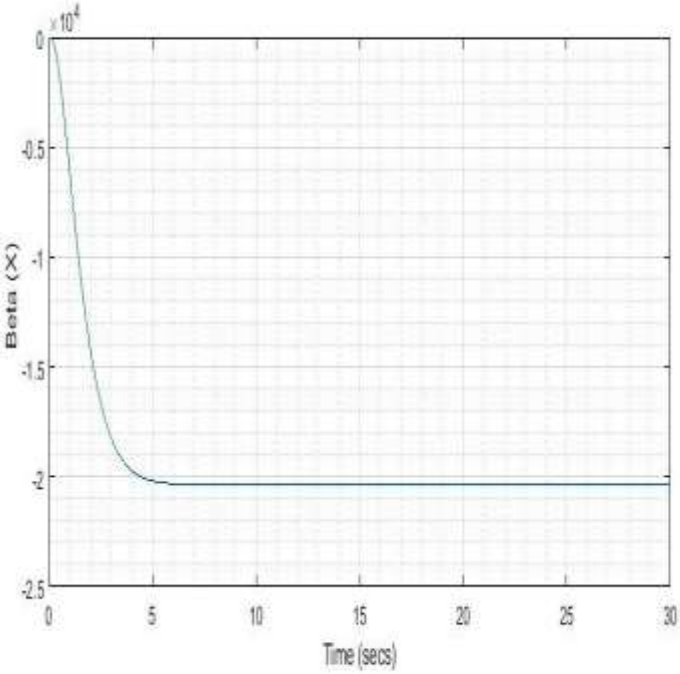

Fig 4b: β_x - Sphere Anglular Displacement in X-direction

In like manner, Fig 4b shows response of the sphere's angular displacement with respect to the x coordinate and attaining stability in 12.8244 seconds.

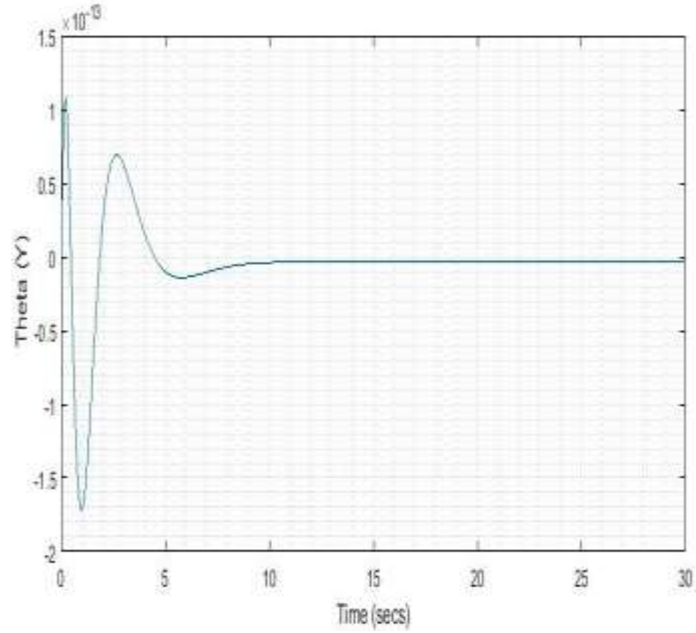

Fig 4c: Θ_y - Ball Angular Displacement in Y-axis

In the y-coordinate, the ball's angular displacement is also stable at 12.8420 seconds after introduction of disturbance, as shown in figure 4c.

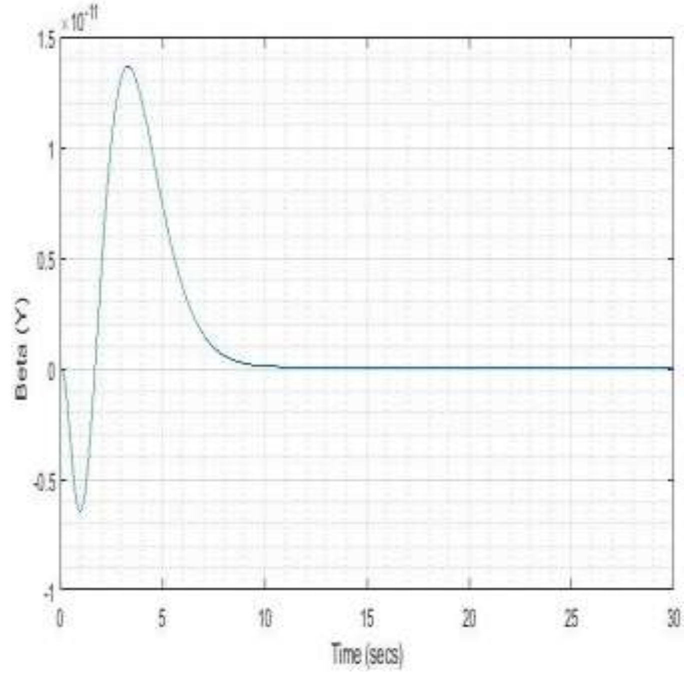

Fig 4d: β_y - Sphere Angular Displacement in Y-direction

As depicted in fig. 4d the angular displacement of the sphere is controlled in 12.8564 seconds.

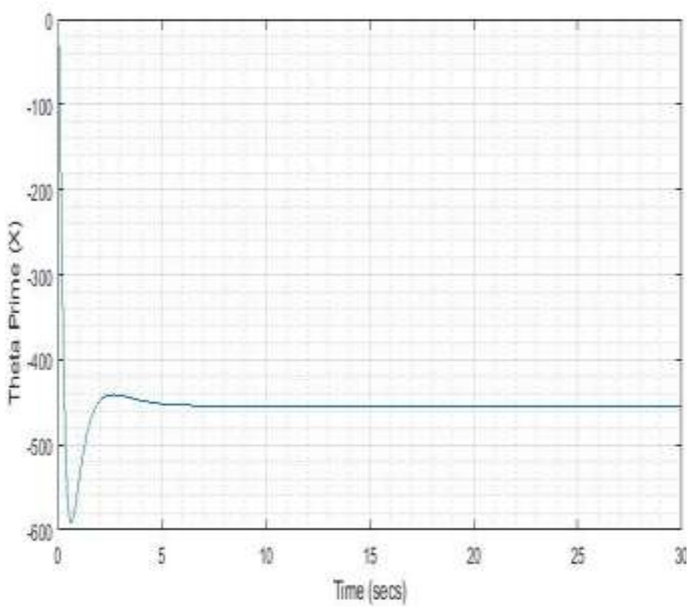

Fig 4e: ϴ´$_x$ - Ball Angular Velocity in X-direction

As for the velocity vector in the x-axis, fig. 4e highlights that the ball's angular velocity velocity was stabilized in 12.8683 seconds under the same disturbance.

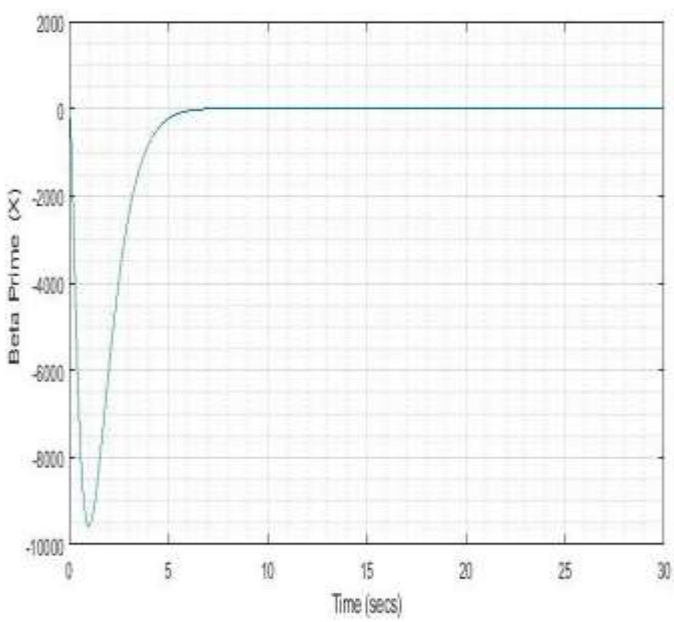

Fig 4f: β´$_x$ - Sphere Angular Velocity in X-axis

The sphere's angular velocity (fig. 4f) exhibited similar response in the x-coordinated by stabilizing at 12.8785 seconds

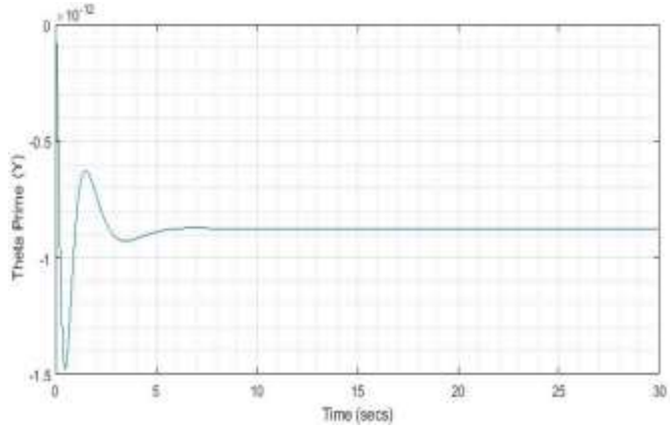

Fig 4g: ϴ´$_y$ - Ball Angular Velocity in Y-direction

Moreover, figs. 4g and 4h shows the ball's and sphere's angular velocities attaining stability in 12.8871 seconds and 12.8947 second respectively amidst same disturbance.

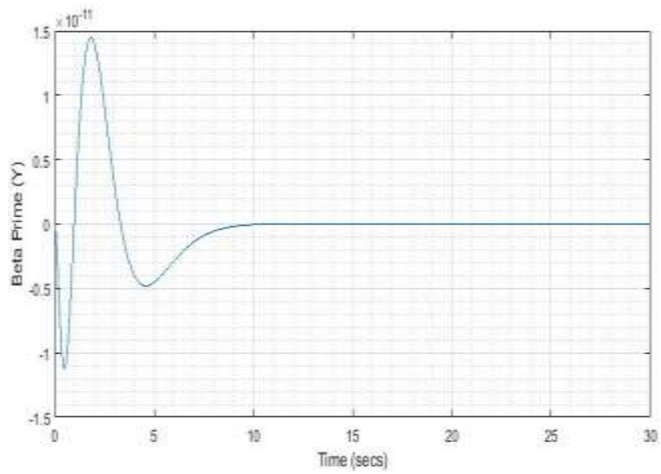

Fig 4h: β´$_y$ - Sphere Angular Velocity in Y-axis

Figures 4a-4h confirms the controllability and observability of the system [17]. Since the model did not consider effects of friction the initial conditions will have negligible influence on the behaviour of the system dynamics [18].

Table 2: Dynamic Response of LQG Control System

Dynamic Response of LQG Control System	**Value (s)**
Ave. Settling Times (secs)	12.85674
Rise Times (secs)	0.74740

On average, Table 2 shows that the LQG controller stabilized the ball at averagely 12.85674 seconds for all the parameters, with no overshoot.

6 CONCLUSION

The control of the BOS through the mechanism of a double feedback loop has been accomplished with a combination of linear algebraic equations and the LQG controller. The simulated results highlights the robust and high performance features of the LQG for controlling the BOS. Moreover, future research work may consider controlling multiple balls on a sphere and adopting artificial intelligent techniques for accomplishing the control problem. Also there is need for further analysis of the system in the frequency domain to give more more insight on system properties such as causal and anti-causal systems; stability; and region of convergence.

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