A SIMULATION STUDY TO DETERMINE THE BEST ESTIMATORS FOR SOLVING PROBLEMS OF AUTOCORRELATION IN LINEAR REGRESSION MODEL

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ABSTRACT

Violation of the assumption of independent explanatory variables and error terms in linear regression model leads to the problems of multicollinearity and autocorrelation respectively. Different estimators that can handle these problems separately have been developed. Moreover, in practice, these two problems do co-exist but estimators to handle them jointly are rare. Consequently, this research proposed and validate two estimators, Feasible Ordinary Ridge Estimators (FORE) and Feasible Generalized Ridge Estimators (FGRE), to handle the problems of autocorrelation separately. The existing and proposed estimators were categorized into five (5) groups namely: One–Stage Estimators (OSE), Two–Stage Estimators (TSE), Feasible Generalized Least Square Estimators (FGLSE), Two-Process Estimators (TPE) and Modified Ridge Estimators (MRE). Monte Carlo experiments were conducted one thousand (1000) times on a linear regression model exhibiting different degrees of multicollinearity ($\lambda =$ 0.4, 0.6, 0.8, 0.95 and 0.99) and autocorrelation ($\rho = 0.4, 0.8, 0.95 \text{ and } 0.99$). However, the multicollinearity in this study is set to zero ($\lambda = 0$). This was examined for both normally and uniformly distributed regressors at sample sizes (n =10, 20, 30, 50, 100 and 250). Finite sampling properties of estimators namely; Bias (BAS), Mean Absolute Error (MAE), Variance (VAR) and most importantly Mean Square Error (MSE) of the estimators were evaluated, examined and compared at each specified level of multicollinearity, autocorrelation and sample size by writing computer programs using Time Series Processor (TSP 5.0) statistical software. These were done by ranking the estimators on the basis of their performances according to the criteria so as to determine the best estimator. With normally distributed regressor, the best estimator is N-AUTOCOFGLSE-ML except at n=10. At this instance, N-AUTOCOFGRE-ML is the best. Also, at sample size of n=20, it is either (N-1)-AUTOCOFGLSE-CORC or OREKBAY that is best. With uniformly distributed regressor, the best estimator is N-AUTOCOFGLSE-ML/ML except at n=50. At this instance, (N-1)-AUTOCOFGLSE-CORC/CORC is the best. Moreover, the GRE and N-AUTOCOFOREKBAY compete at small sample sizes, n=10 and n=20 respectively. Generally, It can be observed from the results that the best estimator is either N-AUTOCOFGLSE-ML/ML or (N-1)-AUTOCOFGLSE-CORC/CORC.

Keywords: Multicollinearity, Autocorrelation, Estimator, Regressors and TSP- Time Series Processor
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INTRODUCTION

The term autocorrelation may be defined as "correlation between members of series of observations ordered in time [as in time series data] or space [as in cross-sectional data].'' In the regression context, the classical linear regression model assumes that such autocorrelation does not exist in the disturbances u_i . Symbolically, $E(u_i u_j) = 0 \quad i \neq j$. The classical model assumes that the disturbance term relating to any observation is not influenced by the disturbance term relating to any other observation. For example, if we are dealing with quarterly time series data involving the regression of output on labor and capital inputs and if, say, there is a labor strike affecting output in one quarter, there is no reason to believe that this disruption will be carried over to the next quarter. That is, if output is lower this quarter, there is no reason to expect it to be lower next quarter. Similarly, if we are dealing with cross-sectional data involving the regression of family consumption expenditure on family income, the effect of an increase of one family's income on its consumption expenditure is not expected to affect the consumption expenditure of another family. However, if there is such a dependence, we have autocorrelation Formby *et al.* [1]. The inefficiency of Ordinary Least Square estimator to estimate the parameters of linear regression model in the presence of autocorrelation led to the development of Generalized Least Squares (GLS) estimator. It requires the true autocorrelation value to be known which is not often so. Using the estimated autocorrelation value leads to the development of Feasible Generalized Least Squares (FGLS) estimators. Thus, studying the finite sample properties of these estimators becomes very imperative. This seems to be very difficult analytically Ayinde and Ipinyomi [2]. However, Monte Carlo approach is often utilized to accomplish this task. Cochrane and Orcutt [3], an economist, observed that the presence of autocorrelated error terms requires some modifications for the OLS estimator to be used. Their suggestion involved an autoregressive transformation of the series involved and that the quasi first differences of such series should be used. Kadiyala [4] observed that the transformation suggested by [3] can lead to a less efficient estimator. He therefore suggested that the addition of one weighted observation to Cochrane Orcutt - CORC procedure may give a better estimator practically without any extra cost. Rao and Griliches [5] did one of earliest Monte Carlo investigations on this study. The estimators they examined include OLS, CORC, Two-Step estimators based on Durbin $\hat{\rho}$ and Prais–Winsten. Their major conclusions were; the OLS estimator is less efficient than all other methods considered for moderate and high values of ($|\rho|>0.3$) and there is a definite gain from using feasible generalized least squares when $|\rho|\geq 0.3$ and little loss from using such methods otherwise. The work of [5] was revisited by Spitzer [6]; but in addition to the estimators considered by [5], he examined the performances of ML estimator. Situations where the two problems exist together in a data set are not uncommon. Therefore, this paper attempts to investigate the performances of the proposed estimators when autocorrelation alone is in the model through Monte Carlo studies.

Though no precise definition of multicollinearity has been firmly established in the literature, multicollinearity is generally agreed to be present if there is an approximate degree of linear relationship among some of the predictor variables in the data. Bowerman and Connell [7] stated that the term multicollinearity refers to a situation in which there is an exact (or nearly exact) linear relation among two or more of the explanatory variables. Exact relations may arise by mistake or lack of understanding. Multicollinearity can also be defined in the concept of orthogonality. When the predictors are orthogonal or uncorrelated, all eigenvalues of the design matrix are equal to one and the design matrix is of full rank. If at least one eigenvalue is different from one, especially when equal to zero or near zero, then non-orthogonality exists, meaning that multicollinearity is present. Multicollinearity can lead to increasing complexity in the research results, thereby posing difficulty for researchers to provide interpretation Charterjee *et al.* [8]. Regardless of whether multicollinearity in data is assumed to be a sampling problem or true reflection of population relationships, it must be looked into when data are analyzed using regression analysis because it has several potential undesirable consequences on the parameter estimates. When multicollinearity is a problem, parameter estimates have wrong signs when compared with theoretical knowledge and variables have insignificant coefficients. The regression coefficients, though determinate when multicollinearity is imperfect, possess large standard errors which imply that the coefficients cannot be estimated with great precision. Hawking and Gujarati [9, 10].

Various other estimation methods have been developed to tackle this problem. These estimators include Stein estimator introduce by James[11], Ridge Regression estimator developed by Hoerl and Kennard [12], Estimator based on Principal Component Regression suggested by Massy[13] and method of Partial Least Squares developed by Wold[14]. Ayinde *et al.* [15] examined the performance estimators based on principal component regression, ML and Cochrane Orcutt.

METHODOLOGY

Estimators for handling autocorrelation

Consider the Linear Regression Model with Autoregressive of order 1, AR (1) given as:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots\ldots\ldots + \beta_k x_{k-t} + u_t \tag{1}$$

Where $u_t = \rho u_{t-1} + \varepsilon_t$

Therefore, the variance – covariance matrix becomes:

$$E(UU') = \sigma^2\Omega$$

$$= \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-2} & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-3} & \rho^{n-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{n-4} & \rho^{n-3} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \rho^{n-2} & \rho^{n-3} & \rho^{n-4} & \ldots & 1 & \rho \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & \rho & 1 \end{bmatrix} \tag{2}$$

and $\sigma^2 = \sigma_u^2 = \frac{\sigma_v^2}{(1-\rho^2)}$,

and the inverse of Ω is given as:

$$\Omega^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \tag{3}$$

We now search for a suitable transformation matrix P^*. If we consider first a (n-1) x n matrix P^* defined as:

$$P^* = \begin{bmatrix} -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ 0 & 0 & -\rho & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \tag{4}$$

By multiplication, it then shows that $P^{*\prime}P^*$ gives an n x n matrix which apart from a proportional constant is identical with Ω^{-1} except for the first elements in the leading diagonal.

Now if we consider an n x n matrix P obtained from P^* by adding a new row to the first row with $\sqrt{1-\rho^2}$ in the first position and zero elsewhere, that is

$$P = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \quad (5)$$

The difference between P^* and P lies only in the treatment of the first sample observation. P^* is much easier to use provided we are prepared to lose information on the first observation. However, when n is large, the difference is negligible, but in small sample, the difference can be major. Moreover, there is need for Ω or more precisely ρ to be known for the GLS via the transformation matrix P^* and P to be used. This is not often the case; we resort to estimating Ω by $\hat{\Omega}$ to have Feasible Generalized Least Squares estimator (FGLSE). There are several ways of consistently estimating ρ, after which either the P^* or P transformation matrix can be used.

The estimators are to handle autocorrelation are: 1. Cochrane and Orcutt Estimator (CORC): Iterative Procedure, 2. Maximum Likelihood Estimator (ML)

Estimators for handling multicollinearity generalized ridge estimator

The generalized ridge estimator of β is given as $\hat{\beta}_k = (X'X + KI)^{-1}X'Y$ where K is a diagonal matrix with non negative diagonal elements $(K_1, K_2, \ldots, K_p)$. The $K_i (i = 1,2,\ldots,p)$ need not be equal. The optimum value of K had been obtained by [12] as:

$$K = \frac{\sigma^2}{\alpha_i^2}\ ;\ i = 1,2,3,\ldots,p. \quad (6)$$

Since σ^2 and α_i^2 are generally unknown, the K_i needs to be estimated. Hoerl *et.al* (1975) suggested the replacement of σ^2 and α_i^2 by their corresponding unbiased estimators $\hat{\sigma}^2$ and $\hat{\alpha}_i^2$. Therefore, $\hat{K}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$ where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-p}$. (The estimated error variance from OLS estimation) and α_i^2 is the regression coefficient from OLS estimation.

The ordinary ridge estimator

The Ordinary ridge regression (ORR) estimator requires a fixed value of the ridge Parameter, K. Several Ks have also been proposed by authors including [12] and that of [16] respectively given as:

$$\hat{K}_{HK} = \frac{\hat{\sigma}^2}{\mathrm{Max}(\hat{\alpha})^2} \quad (7)$$

$$\hat{K}_{LA} = \frac{\hat{\sigma}^2}{[\mathrm{Max}(\hat{\alpha})]^2} \quad (8)$$

Sclove [17] suggested an empirical K-Bayesian Ridge Parameter given as:

$$\hat{K}_{BAY} = \frac{(SSR/n-p)}{(\sum y2\ - SSR/\ \mathrm{trace}(X'X))} \quad (9)$$

Where SSR = Sum of Square of Regression

The proposed estimators:

The proposed estimators were classified into three (3) groups and are presented as follows:

1. **Two–Stage Estimators (TSE):** The discussion requiring an (n x n) or (n-1 x n) non-singular Matrix P for GLSM transformation, the true autocorrelation value (ρ) was used to obtain the Matrix P to transform the data. Thus, the following estimators are proposed. The algorithms required are as follows:

(N-1)-Autocorrelation Corrected Generalized Least Square Estimators [(N-1)-AUTOCOGLSE]:

This estimator requires using (n-1) x n matrix P to transform the data. It can also be referred to as Generalized Least Square Estimator. The procedures are as follows:

i. Transform the data using true autocorrelation value.

ii. Apply OLS estimator on the transformed data to obtain the estimates of the parameters of the regression model.

NOTE: Since these estimators require the use of **true autocorrelation values**, therefore, they do not find relevance in practice.

2. **Two – Process Estimators (TPE):** In Practice, the true autocorrelation value is not known. These estimators are proposed by using the estimated autocorrelation value resulting from known Feasible Generalized Least Square Estimator to transform the data before any other estimator is applied. The estimators and algorithms required are as follows:

3. **Modified Ridge Estimators:** The ridge estimators discussed require estimation of $\hat{K}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$ where,

$\hat{\sigma}^2$ is the Mean Square Error based on OLS estimation

$\hat{\alpha}_i^2$ is the regression coefficient i based on OLS estimation, i = 1,2.........p.

Now, since there is autocorrelation problem, using OLS estimator is inappropriate. It underestimates the variance of the regression co-efficient and the MSE is biased and inconsistent of σ^2 [18]. The proposed estimators used the appropriate estimators, CORC and ML, to obtain MSE and the regression coefficients. These result into the following proposed estimators. The algorithms required are as follows:

Model formulation
Consider the linear regression model given as:
$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \mu_t \qquad (10)$$

Where, $\mu_t = \rho\mu_{t-1} + \varepsilon_{t,}\ |\rho| < 1$,
$t = 1,2,, \dots \dots \dots \dots n, \quad \varepsilon_t \sim N\ (0, \sigma^2)$.
The regressors are fixed and exhibit different degree of multicollinearity.

The Monte Carlo Experiments
The experiment was replicated (R) one thousand time (1000) and at sample sizes of
n = 10,20,30,50,100 and 250.

Generation of Data: Generation of the Explanatory Variables

a. Correlated Normally distributed Variables
The equations provided and used by [19] were used to generate normally distributed random variables with specified inter-correlation. With p = 3, the equations are:
$$X_1 = \mu_1 + \sigma_1 z_1 \qquad (11)$$
$$X_2 = \mu_2 + \lambda_{12}\sigma_{12} + \sqrt{m_{22} z_2} \qquad (12)$$
$$X_3 = \mu_3 + \lambda_{13}\sigma_3 z_1 + \frac{m_{23}}{\sqrt{m_{22}}} + \sqrt{n_{33} z_3} \qquad (13)$$
where,
$m_{22} = \sigma^2(1 - \lambda_{12}^2)$
$m_{23} = \sigma_2\sigma_3(\lambda_{23} - \lambda_{12}\lambda_{13})$
$m_{33} = \sigma^2(1 - \lambda_{13}^2)$
and $n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$; and $z_i \sim N(0,1)$, $i = 1\,,2\,,3$
By these equations, the inter-correlation matrix has to be positive definite among the independent variables.
In the simulation,
$\lambda_{12} = \lambda_{13} = \lambda_{23} = \lambda = 0.4, 0.6, 0.8, 0.95\ and\ 0.99;\ and\ x_i \sim N(0,1)$, $i = 1\,,2\,,3$
The research set (λ=0)

b. Correlated Uniformly Distributed Variables
Using the generated correlated normally distributed variables above, $x_i \sim N(0,1)$, $i = 1\,,2\,,3$; the study further utilized the properties of random variables that cumulative distribution function of normal distribution produce U(0,1) without affecting the correlation among variables to generate correlated uniform distributed variables $x_i \sim U(0,1)$, $i = 1\,,2\,,3$ [20]

c. Generation of the error term
The error terms were generated by using the distributional properties of the autocorrelation error terms of AR (1) model given as:
$$u_t \sim N\left(0, \frac{\sigma_e^2}{(1-\rho^2)}\right) \qquad (14)$$
Thus, assuming the model start from infinite past, the error terms were generated as follows:
$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho^2}} \qquad (15)$$
$$u_t = \rho u_{t-1} + \varepsilon_{t,} \qquad (16)$$
$t = 2\,,3,4\ \dots \dots \dots . n$
In this study, autocorrelation value (ρ) is varied from 0.4, 0.8, 0.95, and 0.99.

d. Generation of Dependent Variable
The model parameter values were taken as $\beta_0 = 0$, $\beta_1 = 0.8$, $\beta_2 = 0.1$ and $\beta_3 = 0.6$. Thus, the dependent variable was also determined. Data were therefore generated for all the specifications of different combinations of multicollinearity, autocorrelation and sample size; a total of one hundred and twenty (120) different combinations (6x5x4) all together.

Estimation methods used in the study
They were categorized into five groups as follows:

1. **One – Stage Estimator (OSE):**
These are existing estimation techniques that require only one stage in their estimation procedures. They are given as follows:
i. Ordinary Least Squares Estimator (OLSE)
ii. Generalized Ridge Estimator (GRE)
iii. Ordinary Ridge Estimator with K – Bayesian (OREKBAY)
iv. Ordinary Ridge Estimator with K- Lukman and Ayinde (OREKLA)

2. **Two -Stage Estimators (TSE):** These are:
i. (N-1)-Autocorrelation Corrected Generalized Least Square Estimators
ii. (N-1) –AUTOCOGLSE]:
iii. (N-1)-Autocorrelation Corrected Generalized Ridge Estimators
[(N-1) –AUTOCOGRE]:
and others remaining estimators

3. **Feasible Generalized Least Square Estimators (FGLSE):** These are existing estimators:
i. Corchran Orcutt Estimator (CORC)
ii. Maximum Likelihood Estimator (ML)

4. **Two- Process Estimators (TPE):**
These are:
i. (N-1) - Autocorrelation Corrected Feasible Generalized Least Square Estimators [(N-1) -AUTOCOFGLSE-CORC]:

ii. (N-1) - Autocorrelation Corrected Feasible Generalized Ridge Estimators [(N-1) -AUTOCOFGRE-CORC]: and others remaining estimators

5. Modified Ridge Estimators (MRE): These are:

i. Cochrane Orcutt Modified Generalized Ridge Estimator (CORCMGRE)

ii. Cochrane Orcutt Modified Ordinary Ridge Estimator-KBAY (CORCMORE-KBAY) and others remaining estimators

Criteria for Evaluation, Examination and Comparison of the estimators

Evaluation, examination and comparison of the estimators were done based on their finite sampling properties. These include Bias closest to zero (BAS), Mean Absolute Error (MAE), Variance (VAR) and Mean Square Error (MSE) defined as follows:

i. $\hat{\beta}_i = \frac{1}{1000}\sum_{j=1}^{1000}\hat{\beta}_{ij}$ (17)

ii.BAS($\hat{\beta}_i$) = $\left|\frac{1}{1000}\sum_{j=1}^{1000}(\hat{\beta}_{ij} - \beta_i)\right|$ (18)

iii.MAE ($\hat{\beta}_i$) = $\frac{1}{1000}\sum_{j=1}^{1000}\left|\hat{\beta}_{ij} - \beta_i\right|$ (19)

iv.VAR($\hat{\beta}_i$)=$\frac{1}{1000}\sum_{j=1}^{1000}(\hat{\beta}_{ij} - \hat{\beta}_i)^2$(20)

v.MSE($\hat{\beta}_i$)=Bias($\hat{\beta}_i$)^2+Var($\hat{\beta}_i$) (21)

RESULTS AND DISCUSSIONS

Results when there is Autocorrelation alone in the model

Samples of results from simulation study and the summary of the best estimator(s) when there is autocorrelation alone in the model are presented as follows:

Samples of results from Simulation Study

Samples of simulation results obtained on the estimators under the mean square error criterion with both normally and uniformly distributed regressors when there is only autocorrelation problem (ρ=0.95) at small sample size (n=10) are given in Table 1

Form Table 1, it can be observed that the results of OREKBAY, GRE, N-AUTOCOFGRE, N-AUTOCOGRE, MLMGRE and MLMOREKLA are the preferred estimators. The estimators N-AUTOCOFGRE-ML and N-AUTOCOFGLSE-ML are the choice estimators with respect to normal and uniform regressors respectively. Similarly, the results of CORC and (N-1)-AUTOCOFGLSE-CORC, ML and N-AUTOCOFGLSE-ML are either the same or approximately the same. With normally distributed regressors, the preferred estimator among the OSE is OREKBAY, among the TSE is N-AUTOCOGRE, among the TPE is N-AUTOCOFGRE-ML, among the FGLSE is ML and among the MRE is MLMGRE or MLMOREKLA. Moreover, with uniformly distributed regressors, the preferred estimator among the OSE is GRE, among the TSE is N-AUTOCOGLSE, among the TPE is N-AUTOCOFGLSE-ML, among the FGLSE is ML and among the MRE is MLMGRE. With normal regressor, the choice estimator is N-AUTOCOFGRE-ML while with uniform regressor the choice estimator is N-AUTOCOFGLSE-ML/ML. The estimators GRE, MLMOREKLA and MLMGRE compete favorably.

Table 1: Mean Square Error of the Estimators when ρ=0.95, λ=0 and n=10

MB1	MB2	MB3	RMB1	RMB2	RMB3	SRMB	ESTIMATOR	CAT	P	N	λ	REGRESSOR
1.53389	0.79363	4.61857	21	24	28	73	OLS	OSE	0.95	10	0	NORMAL
2.32419	0.24476	1.90653	25	4	14	43	GRE	OSE	0.95	10	0	NORMAL
0.77236	0.48474	3.24124	7	12	23	42	**OREKBAY**	OSE	0.95	10	0	NORMAL
0.81695	0.53279	3.21905	10	13	22	45	OREKLA	OSE	0.95	10	0	NORMAL
1.23457	0.7238	0.9652	17	22	4	43	(N-1)-AUTOCOGLSE	TSE	0.95	10	0	NORMAL
0.62908	0.2022	0.75997	2	1	2	5	(N-1)-AUTOCOGRE	TSE	0.95	10	0	NORMAL
0.80058	0.24465	1.071	9	3	8	20	(N-1)-AUTOCOOREKBAY	TSE	0.95	10	0	NORMAL
0.77262	0.28832	1.18647	8	8	9	25	(N-1)-AUTOCOOREKLA	TSE	0.95	10	0	NORMAL
1.16719	0.66823	0.96415	16	18	3	37	N-AUTOCOGLSE	TSE	0.95	10	0	NORMAL
0.58997	0.21089	0.75935	1	2	1	4	**N-AUTOCOGRE**	TSE	0.95	10	0	NORMAL
0.71735	0.25025	1.0479	5	5	6	16	N-AUTOCOOREKBAY	TSE	0.95	10	0	NORMAL
0.66997	0.29151	1.06913	3	9	7	19	N-AUTOCOOREKLA	TSE	0.95	10	0	NORMAL
3.43565	1.14463	2.85627	27.5	25.5	18.5	71.5	(N-1)-AUTOCOFGLSE-CORC	TPE	0.95	10	0	NORMAL
1.38916	0.69053	1.04263	20	21	5	46	(N-1)-AUTOCOFGRE-CORC	TPE	0.95	10	0	NORMAL
0.90138	63.76577	3.83651	13	28	26	67	(N-1)-AUTOCOFOREKBAY	TPE	0.95	10	0	NORMAL
1.60578	0.78417	1.61637	24	23	11	58	(N-1)-AUTOCOFOREKLA	TPE	0.95	10	0	NORMAL
1.26436	0.66848	1.86007	18.5	19.5	12.5	50.5	N-AUTOCOFGLSE-ML	TPE	0.95	10	0	NORMAL
0.77008	0.26508	1.28579	6	6	10	22	**N-AUTOCOFGRE-ML***	TPE	0.95	10	0	NORMAL
3.11847	27.43539	3.59907	26	27	24	77	N-AUTOCOFOREKBAY	TPE	0.95	10	0	NORMAL
0.67526	0.44029	1.94111	4	11	15	30	N-AUTOCOFOREKLA	TPE	0.95	10	0	NORMAL

3.43565	1.14463	2.85627	27.5	25.5	18.5	71.5	CORC	FGLSE	0.95	10	0	NORMAL
1.26436	0.66848	1.86007	18.5	19.5	12.5	50.5	**ML**	FGLSE	0.95	10	0	NORMAL
1.53915	0.36982	2.23012	23	10	17	50	CORCMGRE	MRE	0.95	10	0	NORMAL
0.96817	0.62569	3.8975	15	17	27	59	CORCMOREKBAY	MRE	0.95	10	0	NORMAL
0.96278	0.61286	3.13629	14	16	21	51	CORCMOREKLA	MRE	0.95	10	0	NORMAL
1.53415	0.26674	2.03878	22	7	16	45	**MLMGRE**	MRE	0.95	10	0	NORMAL
0.88871	0.58627	3.7206	12	15	25	52	MLMOREKBAY	MRE	0.95	10	0	NORMAL
0.83055	0.56629	2.96164	11	14	20	45	**MLMOREKLA**	MRE	0.95	10	0	NORMAL
0.7189	0.17482	0.30935	28	28	28	84	OLS	OSE	0.95	10	0	UNIFORM
0.32784	0.097888	0.18005	16	22	16	54	**GRE**	OSE	0.95	10	0	UNIFORM
0.36202	0.093734	0.20907	18	21	21	60	OREKBAY	OSE	0.95	10	0	UNIFORM
0.37873	0.10677	0.19928	22	23	20	65	OREKLA	OSE	0.95	10	0	UNIFORM
0.10719	0.054859	0.076749	2	2	2	6	(N-1)-AUTOCOGLSE	TSE	0.95	10	0	UNIFORM
0.36438	0.065795	0.17404	19	8	15	42	(N-1)-AUTOCOGRE	TSE	0.95	10	0	UNIFORM
0.29488	0.070701	0.16234	9	11	11	31	(N-1)-AUTOCOOREKBAY	TSE	0.95	10	0	UNIFORM
0.29643	0.076136	0.1624	10	17	12	39	(N-1)-AUTOCOOREKLA	TSE	0.95	10	0	UNIFORM
0.10648	0.054765	0.072952	1	1	1	3	**N-AUTOCOGLSE**	TSE	0.95	10	0	UNIFORM
0.37128	0.062971	0.15564	20	4	10	34	N-AUTOCOGRE	TSE	0.95	10	0	UNIFORM
0.30604	0.067545	0.14509	12	9	5	26	N-AUTOCOOREKBAY	TSE	0.95	10	0	UNIFORM
0.30888	0.072598	0.14567	14	12	6	32	N-AUTOCOOREKLA	TSE	0.95	10	0	UNIFORM
0.16557	0.073339	0.16431	5.5	13.5	13.5	32.5	(N-1)-AUTOCOFGLSE-CORC	TPE	0.95	10	0	UNIFORM
0.3403	0.07641	0.19675	17	18	18	53	(N-1)-AUTOCOFGRE-CORC	TPE	0.95	10	0	UNIFORM
0.30739	0.075532	0.18922	13	16	17	46	(N-1)-AUTOCOFOREKBAY	TPE	0.95	10	0	UNIFORM
0.31002	0.082219	0.19775	15	19	19	53	(N-1)-AUTOCOFOREKLA	TPE	0.95	10	0	UNIFORM
0.11856	0.065066	0.079896	3.5	6.5	3.5	13.5	**N-AUTOCOFGLSE-ML***	TPE	0.95	10	0	UNIFORM

0.30148	0.059949	0.15269	11	3	9	23	N-AUTOCOFGRE-ML	TPE	0.95	10	0	UNIFORM
0.25674	0.064064	0.14729	8	5	7	20	N-AUTOCOFOREKBAY	TPE	0.95	10	0	UNIFORM
0.25184	0.069626	0.15109	7	10	8	25	N-AUTOCOFOREKLA	TPE	0.95	10	0	UNIFORM
0.16557	0.073339	0.16431	5.5	13.5	13.5	32.5	CORC	FGLSE	0.95	10	0	UNIFORM
0.11856	0.065066	0.079896	3.5	6.5	3.5	13.5	**ML**	FGLSE	0.95	10	0	UNIFORM
0.38711	0.083424	0.24345	23	20	23	66	CORCMGRE	MRE	0.95	10	0	UNIFORM
0.65463	0.15811	0.28969	27	27	27	81	CORCMOREKBAY	MRE	0.95	10	0	UNIFORM
0.54365	0.14198	0.26347	25	25	25	75	CORCMOREKLA	MRE	0.95	10	0	UNIFORM
0.37225	0.073944	0.23863	21	15	22	58	**MLMGRE**	MRE	0.95	10	0	UNIFORM
0.64785	0.15681	0.28636	26	26	26	78	MLMOREKBAY	MRE	0.95	10	0	UNIFORM
0.51382	0.13794	0.25227	24	24	24	72	MLMOREKLA	MRE	0.95	10	0	UNIFORM

Notes: (i) Preferred Estimator is bolded
(ii) Choice estimator is bolded and asterisked

Table 2: Summary of the best estimators on the basis of criteria at different sample sizes with normally and uniformly distributed regressors when autocorrelation alone is in the model

	NORMAL					UNIFORM				
N	**BIAS**	**MAE**	**VAR**	**MSE**	**OVERALL**	**BIAS**	**MAE**	**VAR**	**MSE**	**OVERALL**
10	**(N-1)-AUTOCOFGLSE-CORC/CORC (1)** **N-AUTOCOFGLSE-ML/ML (1)** **MLMORE KLA (1)** **CORCMOREKLA(1)**	GRE (1) **N-AUTOCOFGRE-ML (3)**	GRE (1) **N-AUTOCOFGRE-ML (3)**	GRE (1) **N-AUTOCOFGRE-ML (3)**	(N-1)-AUTOCOFGLSE-CORC/CORC (1) N-AUTOCOFGLSE-ML/ML (1) MLMORE KLA (1) CORCMOREKLA(1) **N-AUTOCOFGRE-ML (9)** GRE (3)	OLSE (1) **N-AUTOCOFGLSE-ML/ML (2)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	MLMORE-KLA (1) **N-AUTOCOFGLSE-ML/ML (3)**	GRE (1) **N-AUTOCOFGRE-ML (3)**	GRE (1) **N-AUTOCOFGLSE-ML/ML (3)**	OLSE (1) **N-AUTOCOFGLSE-ML/ML (8)** (N-1)-AUTOCOFGLSE-CORC/CORC (1) N-AUTOCOFGRE-ML (3) GRE (2) MLMORE-KLA (1)
20	OLSE (1) **N-AUTOCOFGLSE-ML/ML (3)**	**OREKBAY (2)** **(N-1)-AUTOCOFGLSE-CORC/CORC (2)**	OREKBAY (1) **(N-1)-AUTOCOFGRE-CORC (2)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	**OREKBAY (2)** **(N-1)-AUTOCOFGLSE-CORC/CORC (2)**	N-AUTOCOFGLSE-ML/ML (3) **(N-1)-AUTOCOFGLSE-CORC/CORC (5)** OLSE (1) **OREKBAY (5)** (N-1)-AUTOCOFGRE-CORC	OLSE (1) **(N-1)-AUTOCOFGLSE-CORC/CORC (2)** N-AUTOCOFGLSE-ML (1)	N-AUTOCOFORE-KBAY (1) **N-AUTOCOFGLSE-ML/ML (3)**	N-AUTOCOFORE KBAY (1) **N-AUTOCOFGRE-ML (3)**	N-AUTOCOFORE KBAY (1) **N-AUTOCOFGLSE-ML/ML (3)**	OLSE (1) (N-1)-AUTOCOFGLSE-CORC/CORC (2) **N-AUTOCOFGLSE-ML (7)** N-AUTOCOFORE-KBAY (3)

					(2)					N-AUTO COFGRE-ML (3)
30	**N-AUTO COFGLSE-ML**/ML **(2) (N-1)-AUTO COFGLSE-CORC/ CORC (2)**	OREKBAY (1) **N-AUTO COFGLSE-ML/ML (2)** (N-1)-AUTO COFGLSE-CORC/ CORC (1)	OREKLA (1) **N-AUTO COFGRE-ML (2)** N-AUTO COFGLSE-ML/ML (1)	OREKBAY (1) **N-AUTO COFGLSE-ML/ML (2)** (N-1)-AUTO COFGLSE-CORC/ CORC (1)	**N-AUTO COFGLSE-ML**/ML **(7)** (N-1)-AUTO COFGLSE-CORC/ CORC (4) OREKBAY (2) OREKLA (1) N-AUTO COFGRE-ML (2)	**(N-1)-AUTO COFGLSE-CORC/ CORC (4)**	**N-AUTO COFGLSE-ML/ML (2) (N-1)-AUTO COFGLSE-CORC/ CORC (2)**	(N-1)-AUTO COFORE KBAY (1) **(N-1)-AUTO COFGRE-CORC (3)**	**N-AUTO COFGLSE-ML/ML (2) (N-1)-AUTO COFGLSE-CORC/ CORC (2)**	N-AUTO COFGLSE-ML/ML (4) **(N-1)-AUTO COFGLSE-CORC/ CORC (8)** (N-1)-AUTO COFORE KBAY (1) (N-1)-AUTO COFGRE-CORC (3)
50	**(N-1)-AUTO COFGLSE-CORC/** CORC **(3)** N-AUTO COFGLSE-ML (1)	N-AUTO CO FORE-KBAY (1) N-AUTO COFGLSE-ML/ML **(1) (N-1)-AUTO COFGLSE-CORC/** CORC **(2)**	N-AUTOCO FORE-KBAY (1) N-AUTO COFGLSE-ML/ML (1) **(N-1)-AUTO COFGRE-CORC (2)**	OREKBAY (1) **N-AUTO COFGLSE-ML/ML (2)** (N-1)-AUTO COFGLSE-CORC/ CORC (1)	**(N-1)-AUTO COFGLSE-CORC/ CORC (8)** N-AUTO COFGLSE-ML (5) N-AUTOCO FORE-KBAY (2) OREKBAY (1)	**N-AUTO CO FGLSE-ML/ML (3)** CORC (1)	**N-AUTO COFGLSE-ML/ML (3)** (N-1)-AUTO COFGLSE-CORC/ CORC (1)	N-AUTO CO FGRE-ML (1) **(N-1)-AUTO COFGRE-CORC (2)** (N-1)-AUTO COFGLSE-CORC (1)	N-AUTO COFGLSE-ML/ML (1) **(N-1)-AUTO COFGLSE-CORC/ CORC (3)**	**N-AUTO COFGLSE-ML/ML (7)** (N-1)-AUTO COFGLSE-CORC/ CORC (6) N-AUTO CO FGRE-ML (1) (N-1)-AUTO COFGR

										E-CORC (2)
100	**(N-1)-AUTOCOFGLSE-CORC/CORC (2)** N-AUTOCOFGLSE-ML/ML (2)	**N-AUTOCOFGLSE-ML/ML (3)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	ML/MLMGRE (1) **N-AUTOCOFGLSE-ML/ML (2)** (N-1)-AUTOCOFGRE-CORC/CORC (1)	OREKBAY (1) **N-AUTOCOFGLSE-ML/ML (2)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	(N-1)-AUTOCOFGLSE-CORC/CORC (5) **N-AUTOCOFGLSE-ML/ML (9)** ML/MLMGRE (1) OREKBAY (1)	**N-AUTOCOFGLSE-ML/ML (2)** **(N-1)-AUTOCOFGLSE-CORC/CORC (2)**	**N-AUTOCOFGLSE-ML/ML (2)** **(N-1)-AUTOCOFGLSE-CORC/CORC (2)**	**(N-1)-AUTOCOFGRE-CORC (4)**	**N-AUTOCOFGLSE-ML/ML (3)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	N-AUTOCOFGLSE-ML/ML (7) **(N-1)-AUTOCOFGLSE-CORC/CORC (9)**
250	**N-AUTOCOFGLSE-ML/ML (2)** **(N-1)-AUTOCOFGLSE-CORC/CORC (2)**	**N-AUTOCOFGLSE-ML/ML (3)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	**N-AUTOCOFGRE-ML (2)** N-AUTOCOFGLSE-ML/ML (1) (N-1)-AUTOCOFGLSE-CORC/CORC (1)	**N-AUTOCOFGLSE-ML/ML (3)** (N-1)-AUTOCOFGLSE-COC/CORC (1)	**N-AUTOCOFGLSE-ML/ML (9)** (N-1)-AUTOCOFGLSE-CORC/CORC (5) N-AUTOCOFGRE-ML (2)	**N-AUTOCOFGLSE-ML/ML (3)** (N-1)-AUTOCOFGLSE-CORC/CORC (1)	**N-AUTOCOFGLSE-ML/ML (4)**	N-AUTOCOFGRE-ML (1) **(N-1)-AUTOCOFGRE-CORC (3)**	**N-AUTOCOFGLSE-ML/ML (4)**	**N-AUTOCOFGLSE-ML/ML (11)** (N-1)-AUTOCOFGLSE-CORC/CORC (1) N-AUTOCOFGRE-ML (1) (N-1)-AUTOCOFGRE-CORC (3)

From Table 2, the following are observed about the best estimators under Mean Square Error criteria.

Mean Square Error

i. With normally distributed regressor, the best estimator is N-AUTOCOFGLSE-ML except at n=10. At this instance, N-AUTOCOFGRE-ML is the best. Also, at sample size of n=20, it is either (N-1)-AUTOCOFGLSE-CORC or OREKBAY that is best. Also, at various sample sizes, the (N-1)-AUTOCOFGLSE-CORC does compete with the best estimator. This is presented pictorially in Figure 1A.

ii. With uniformly distributed regressor, the best estimator is N-AUTOCOFGLSE-ML/ML except n=50. At this instance, (N-1)-AUTOCOFGLSE-CORC/CORC is the best. Moreover, the GRE and N-AUTOCOFOREKBAY compete at small sample sizes, n=10 and n=20 respectively. The frequency of the choice estimators at various sample sizes is presented pictorially in Figure 1B.

The best estimator is either N-AUTOCOFGLSE-ML/ML or (N-1)-AUTOCOFGLSE-CORC/CORC.

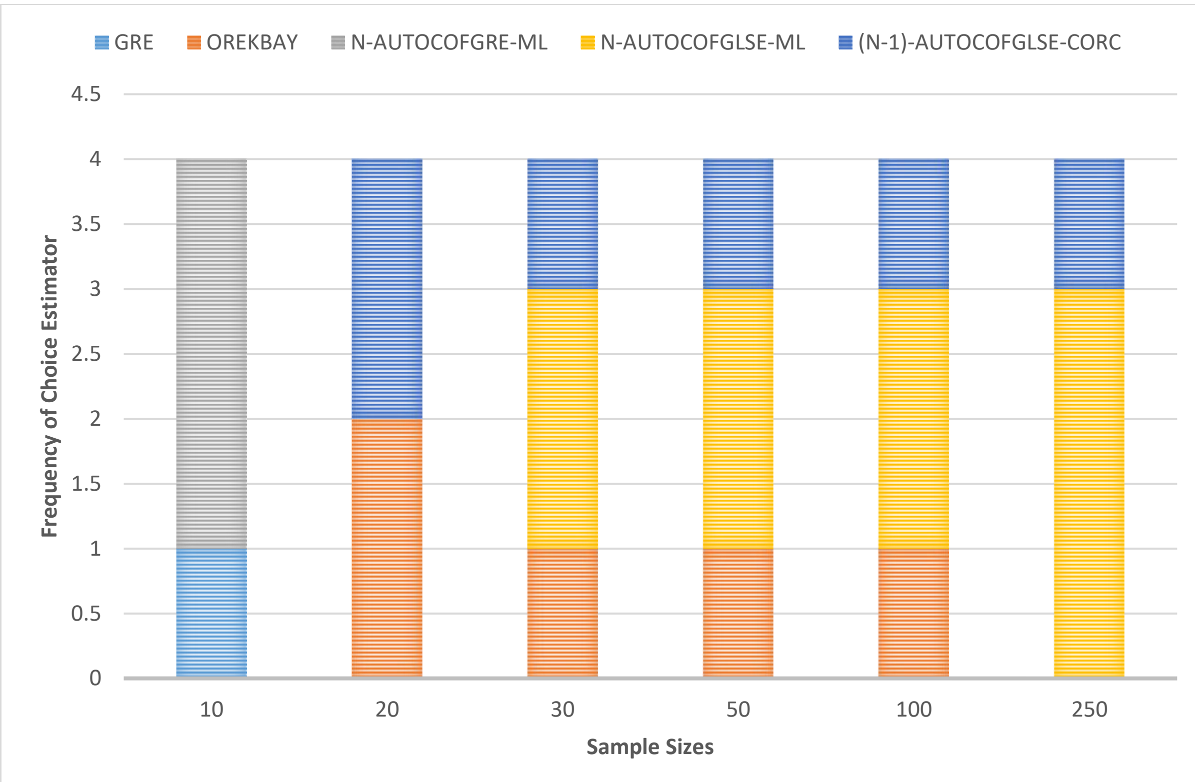

Figure 1A: Compound Bar Chart of Choice Estimators with Normally Distributed Regressors when Autocorrelation alone is in the model (MSE Criterion)

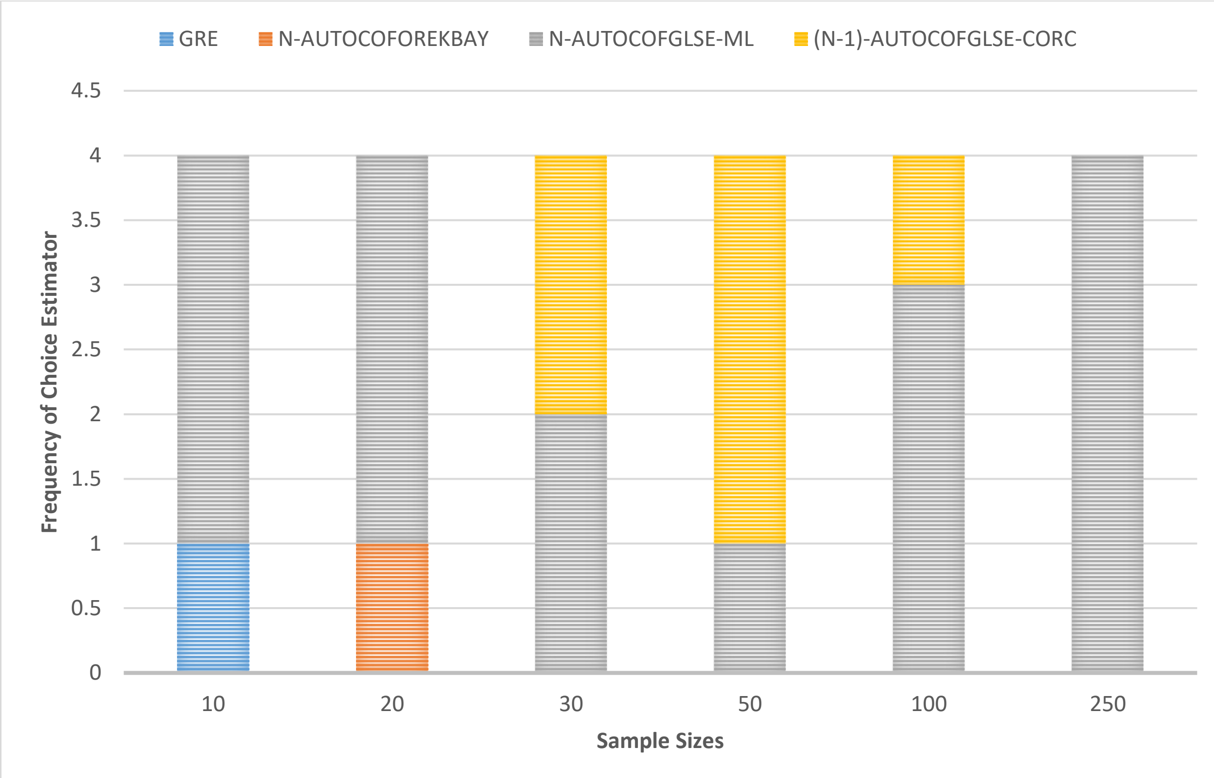

Figure 1B: Compound Bar Chart of Choice Estimators with Uniformly Distributed Regressors when Autocorrelation alone is in the model (MSE Criterion)

CONCLUSION

It can be observed from the results that best estimator is either N-AUTOCOFGLSE-ML/ML or (N-1)-AUTOCOFGLSE-CORC/CORC. These results support the findings by Rao and Grilliches (1969), Ayinde and Ipinyomi (2007) and Ayinde, Lukman and Arowolo (2015). With normally distributed regressor, the best estimator is N-AUTOCOFGLSE-ML except at n=10. At this instance, N-AUTOCOFGRE-ML is the best. Also, at sample size of n=20, it is either (N-1)-AUTOCOFGLSE-CORC or OREKBAY that is best. With uniformly distributed regressor, the best estimator is N-AUTOCOFGLSE-ML/ML except at n=50. At this instance, (N-1)-AUTOCOFGLSE-CORC/CORC is the best. Moreover, the GRE and N-AUTOCOFOREKBAY compete at small sample sizes, n=10 and n=20 respectively.

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