DEVELOPMENT OF A MODIFIED SIMPLE EVENT FUNCTION FOR EVENT-BASED CONTROL OF INVERTED PENDULUM ON A CART

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ABSTRACT

This paper presents the development of a modified simple event function (mSEF) for event-based control of inverted pendulum (IP) on a cart. The simple event function (SEF) control is an event-based control which updates control law at the occurrence of an event. This control scheme is aimed at minimizing system resource utilization and computational time. However, there is the issue of performance degradation associated with this control scheme. Hence the SEF was modified by introducing the time derivative of the last event state into the event function, which established a better trade-off between computational time and control system performance. The result obtained from simulation shows that the control system met all the design specifications. A rise time of 0.721 seconds was recorded as against the specified 0.9 seconds. The pendulum angle also did not exceed the specified 0.35 radians from the vertical. The system settled at 3.742 seconds which is less than the specified 5.0 seconds. No performance degradation was noticed beyond the settling time. The modified simple event function control was able to hold the inverted pendulum at its vertical position after settling. All simulations were carried out using MATLAB/Simulink 2015a.

1. INTRODUCTION

In Control and Signal processing, rapid development in digital computers in recent years has been a game changer in the field. Research geared towards discrete-time systems have recently increased due to advancements in microprocessor technology. Microprocessors are now smaller and efficient in terms of cost, speed, heat dissipation and energy usage unlike the earlier versions [1]. Discrete time based control systems are basically sample systems periodically with a constant period of time T, which computes and updates the control effort for every $t_k = KT$ [2]. Sampling is achieved theoretically on a central periodic interval generated by a system clock and which controls sampling, computing and actuation. For discrete time systems, systems are sampled only at discrete time intervals. One of the important challenge is selecting the best-fit sampling interval for the system. Popularly used is the time driven control method where samples are taken periodically and at equal distance from each other. This control method is simple to design and implement making it a preferred technique for most control systems. More so, there have been well developed theories on the application of this strategy and the results have been proven to yield considerable system satisfaction. However, miniature integrated systems are constrained in the area of the size of power modules available thereby limiting the energy available. It is important therefore to optimize the energy utilization of integrated systems in order to increase system availability [1]. Also, in large scale distributed control system, where digital communication network is used to implement the data flow between sensors, controllers and actuators, resource utilization has been an issue. Resource-aware design is a popular scheme used to effectively design the efficient usage of resources in distributed control systems. This is done with the view of making use of resources in such a way that the resources are adequately put to use in the best way possible. These control schemes are aimed at improving system availability and performance and also to optimize the utilization of system resources. These challenges led to the reformulation of answers to basic questions: when to sample and update control effort or when is it best to communicate [3]. Event-based sampling and control algorithm was designed out of the necessity to address various control systems objectives. One of the major objectives that the event-based control was designed to achieve was to update control effort only when needed in order to attain the desired system performance. Event-based also called event-triggered control has gained attention from researchers in recent time due to its promises of better system performance and resource utilization. Considering this control scheme, events trigger the execution of control task. Sampling time can be explained as the interval that exists between consecutive events [4].

In control systems, a typical Event-based feedback consists mainly of two functions which are:

1. **Event Function (e)** this function decides if there is need or not to re-calculate and update control signal. Event function e denotes the value of weighted error of the absolute difference between the current state of the system and the state of the system as at the last event. Usually e is termed a threshold. Event function that uses the system current state are memoryless. The regularity of e has no contraint.
2. **System Feedback Function (k)**. This is the feedback control law that will be calculated and updated once an event occurs or becomes true.

There are numerous advantages that the event-based strategies offer. For example, in event control scheme, system outputs are changed at the occurrence of events activated by conditions that are met. This among other advantages helps to prolong the life cycle of these actuators. One of the advantage of this scheme is that it achieves optimized energy utilization and acceptable system performance [1]. More so, challenges in networked systems can be solved using event control scheme [5]. In traditional periodic sampling, samples are taken at fixed time interval as against being activated by events in event control scheme. For example, update of control effort is reduced when event based control is implemented which means increased resources availability. In network system, this translates to reduction in traffic, delays and message drops. Consequently, event driven control scheme has the potential of improving battery life of devices in remote locations as a result of reduced resource usage. In mechanical systems, wear and tear on parts due to periodic and regular update of control law is reduced because of fewer control signal update, thereby improving the life cycle of the actuators. Compared to periodic control, event driven control has shorter latencies which means it need not to wait till the next period before control is updated [6].

Several efficient methods for event-based control over the last decades have been researched such as send-on-delta/Lebesgue sampling [7]-[9], send-on-area [10], or send-on-energy schemes [3], event triggered control [11]-[12], self-triggered control [13]-[14], stability can be achieved using event-driven control based on tracking variations of Lyapunov function which stands for the total energy of the system (Lyapunov sampling) [15], periodic event triggered control [16]-[17], intermittent control [18]. However, applying event driven scheme in non-linear systems, calculating the event function employs the use of more resources than calculating the control function because of a Lyapunov function which might not be available [4]. To this effect, simple event function control was proposed which involves equating a tolerance bound to the difference of the current state and the state at the last event. The problem with the simple event is that it does not perform too well when disturbances are introduced. The proposed modified simple event function control is aimed at minimizing computational cost of the event driven controller while achieving a similar performance. Inverted pendulum on a cart is a pendulum whose mass is above its pivot point and attached to a movable cart. It is inherently unstable and has to be actively balanced in order to remain upright and resistant to disturbance [15]. The control of the inverted pendulum constitutes a major research area in the field of engineering and resembles existing control systems such as found in several fields and industries: medicine, robotics, aviation, and defence. It represents a class of non-linear control systems and can be said to be a very useful problem in understanding fundamental control schemes. The inverted pendulum is also useful in research and learning purposes because of its popularity in applications such as human transporter segway (HTS), the prosthetic knee which resembles a double inverted pendulum, the humanoid robots, missile launchers, pendulous and several other implementations notably applied in constructing anti-earthquake buildings. International Federation of Automatic Control (IFAC) regularly compares control schemes in order to benchmark control challenges. A cart-inverted pendulum is such a benchmark control problem [19]. The inverted pendulum system which is a multi-variable, under-actuated, perturbed, strong coupling and unstable system is a favorite experimental objective in control system [20]. Inverted pendulum is a nonlinear and unstable system used as test bench to implement and validate control schemes in the field of Engineering [21]. The objective of the control scheme is to either swing up or stabilize the pendulum near its inverted position. A force is applied to the cart to move in the x-axis in order to control the pendulum angle and prevent it from falling over. Event driven control has been implemented to achieve the control objective in the past [22]. More so, in the classical approach, a dynamical state feedback control that calculated the linearized model of the system was applied to control the inverted pendulum like in [23]-[25]. The proposed modified simple event function control will be used to stabilize the inverted pendulum around its inverted position.

1.1 Implementing of the Simple Event Function Control

1. Define the state space matrices A, B, C, D of the inverted pendulum system
2. Calculate the closed loop feedback gain of the controller
3. Define the simple event function and the tolerance bound $\bar{e}$
4. Create and simulate the Simulink model of the closed loop system
5. Display the results

1.2 Modification of the Simple Event Function

1. Define the simple event function
2. Define the time derivative of the state at the last event time
3. Introduce the time derivative of the state at the last event time to the simple event function.
4. Develop the tolerance bound $\bar{e}$ *and* $\bar{a}$ using the minimum sampling interval

1.3 Imlementation of the Modified Simple Event Function Control

1. Repeat steps A (1) and A (2)
2. Define the modified simple event function and tolerance bounds $\bar{e}$ and $\bar{a}$
3. Repeat steps A (4) and A (5)

2. DISCUSSION OF RESULTS

Modified simple event function control was used to stabilize the inverted pendulum on a cart system in response to a step input. The control system is expected to meet the control system design requirements as follows:

The design requirements are [2]:

1. Settling time for pendulum angle φ of less than 5 seconds
2. Rise time of 0.9 seconds
3. Pendulum angle φ never more than 0.35 radians from the vertical
4. Steady state error of less than 2 % for the pendulum angle φ

A plot of the system states against time and time scope for the pendulum angle were reported in Figure 1 and Figure 2 respectively.

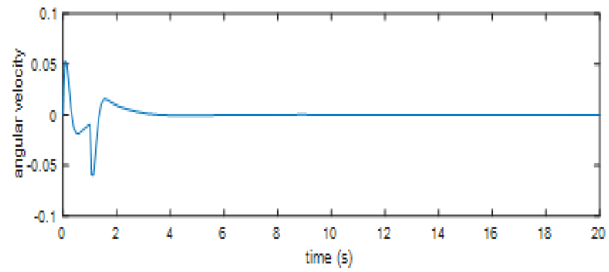

Figure 1a: Plot of pendulum angular velocity

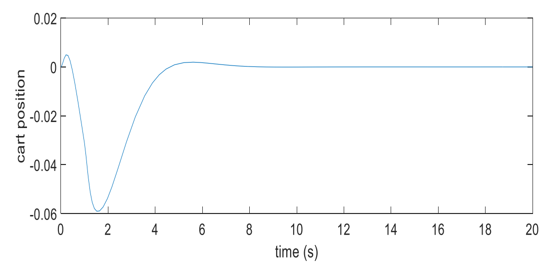

Figure 1b: Plot of cart position

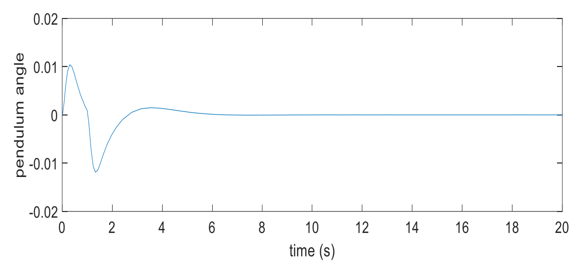

Figure 1c: Plot of pendulum

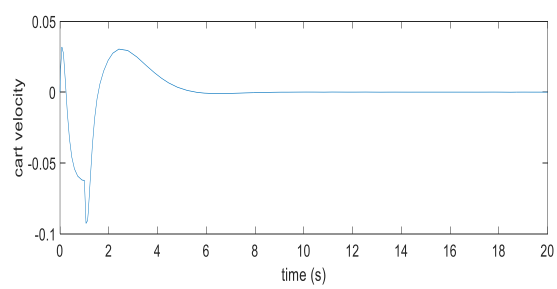

Figure 1d: Plot of cart velocity

Figure 1(a-d): Represent the plot of the system states against time using mSEF control

From Figure 1(a), it can be observed that the controller was not aggressive in moving the pendulum angle to its upward position. More so, Figure 1(c), shows the cart position which reveals that the controller action was smooth and the cart did not move too far away from its starting position. The controller was able to stabilize the inverted pendulum around its upward position without degradation.

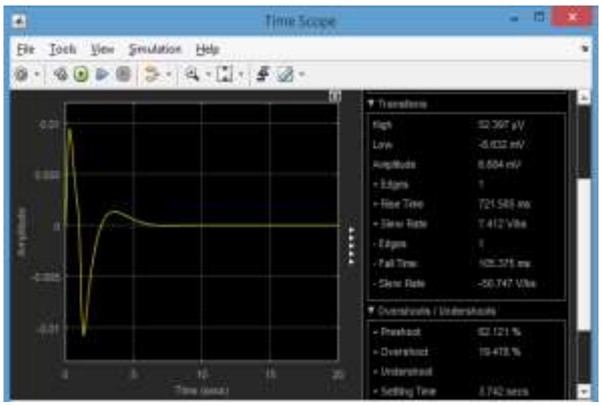

Figure 2: Time scope graph for pendulum angle using mSEF control

From Figure 2, the modified simple event function control was able to stabilize the inverted pendulum around its vertical position. The control system met all the design specifications set in section 3.2. A rise time of 0.721 seconds was recorded as against the specified 0.9 seconds. The pendulum angle also did not exceed the specified 0.35 radians from the vertical. The system settled at 3.742 seconds which is less than the specified 5.0 seconds. No performance degradation was noticed beyond the settling time. The modified simple event function control was able to hold the inverted pendulum at its vertical position after settling.

3. CONCLUSION AND RECOMMENDATION FOR FURTHER WORK

The mSEF control was developed by introducing the time derivative of the system state at the last event which eliminated the problem of performance degradation of the SEF control. The tolerance bounds $\bar{e}$ *and* $\bar{a}$ for the mSEF was developed using the MSI which ensured a non-zero consecutive control law update. The mSEF control was implemented in MATLAB/Simulink 2015a and the performance of the mSEF control was evaluated using the inverted pendulum on a cart. The mSEF control stabilized the inverted pendulum in the inverted position within the specified control system design requirement. The simulation results obtained showed that mSEF control had a good performance in stabilizing the inverted pendulum. The following possible areas of further work are recommended for consideration for future research:

a. Application of the developed control scheme on systems with disturbances to determine the performance and disturbance rejection property of the control scheme.

b. Modification of the developed scheme to further reduce the computational time and improve its performance in relation to settling time.

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