

## IMPACT OF NUMBER OF CHANNELS ON THE CAPACITY OF SINGLE AND MULTI-CHANNEL WIRELESS NETWORKS



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### ABSTRACT

*Radio Frequency (RF) suffers lot of impairments and perturbations resulting in counters such as call drops rate, failure rate and congestion rate on one hand and beam-width, fading, shadowing and attenuation on the other hand. These factors seriously affect systems capacity. To combat the effect of these impairments on the signal, the information can be transmitted via multiple channels by deploying multiple antennas at both transmitter and receiver. This antenna configuration is called Multiple-Input-Multiple-Output (MIMO). In this paper, the performance of transmission over single channel was first investigated, modelled and compared with that of multiple channels. Also, a mathematical model based on Shannon capacity formula was used to obtain the capacity of different channels for fair comparison. The results obtained shows that, the capacity from the coherent channel is 1.246 times higher than that of single channel. Also, that of non-coherent channel is 1.122 times higher than that of single channel. Using the same index, orthogonal channel gives an average capacity of  $n/2$  higher than that of single antenna, this implies that, for a given number of antennas say eight ( $n=8$ ) at the same signal to noise ratio, the capacity of orthogonal channel is about four (4) times higher than that of single channel. The results indicate that the throughput and the reliability of the link improved more significantly when the information is being sent over multiple uncorrelated channels. This can be achieved by the use of orthogonal MIMO channels.*

## 1. INTRODUCTION

The increasing number of wireless users and applications has placed new demands on capacity in the radio networks. The network is required not only to offer services like voice and SMS, but also internet access and streaming video. Applications of this kind call for bit rates significantly higher than those of second-generation systems [1].

To fulfil these requirements, First and second generations of mobile technologies could not provide these demands [2]. The third generation of mobile communication system came into play, which made a considerable improvement in data transmission by supporting multimedia services like video-on-demand and internet access [3]. The evolutions continue until the recently proposed 5G technology. The goal of this

evolutions is to encourage ease access to mobile communication services at low cost and high quality.

However, the biggest problems in wireless networks is how to offer a robust communication over fading channels. Spatial diversity, in the form of employing multiple antennas at both the transmitters end and the receivers end known as multiple input multiple output (MIMO) has been proposed in [4] This methods prove to be effective in combatting fading in wireless channel. According to [5], equipping a wireless mobile device with multiple antennas is quite challenging. But, recent studies, particularly on antenna design provided highly compact prototype which could seamlessly be fitted in handheld mobile devices.

The ability to provide high data rate and improved capacity is a key measure for the current 5G

technology. Increasing data rates can be achieved by transmitting multiple parallel streams to a single user via deploying multiple antennas at both transmitter and receiver. This antenna configuration is called Multiple-Input-Multiple-Output (MIMO) [6]. As shown in Figure 1, there are four antenna configurations in wireless systems: SISO (Single-Input-Single-Output), SIMO (Single-Input-Multiple-Output), MISO (Multiple-Input-Single-Output) and MIMO (Multiple-Input-Multiple-Output).

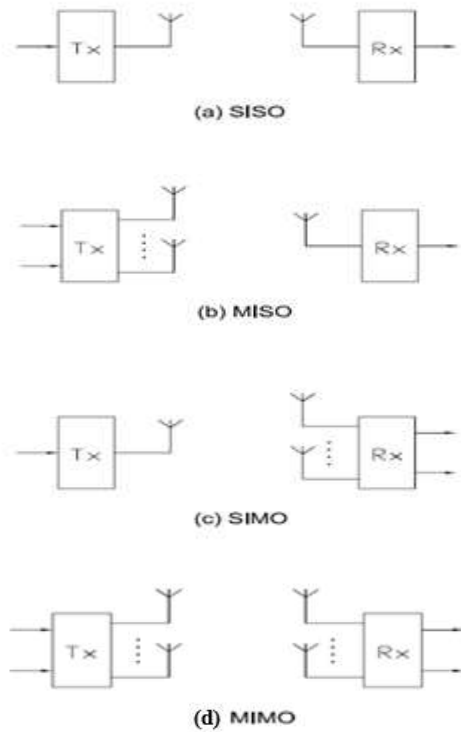


Figure 1: Antenna Configurations

Many approaches such as interference cancellation, smart antenna, and fast transmit power control (TPC), cooperative MIMO relaying, etc. are being actively studied to reduce the interference from other users and hence increase the link capacity [8].

In this paper, both SISO and MIMO channels are modelled and the performance of transmission over the single channel was first investigated and compared with that of multiple channels. Also, a mathematical model based on Shannon capacity formula was used to obtain the capacity of different channels.

## 2. MIMO SYSTEM MODEL

For a single point-to-point MIMO system with arrays of  $n_T$  transmit and  $n_R$  receive antennas with a complex baseband linear system model described in discrete time. As shown in Figure 2, the transmitted signals in each symbol period are represented by an  $n_T \times 1$  column matrix  $x$ , where the  $i$ th component  $x_i$ , refers to the transmitted signal from the  $i$ -th antenna. Considering a Gaussian channel, for which, according to information theory [7], the optimum distribution of transmitted signals is also Gaussian. Thus, the elements of  $x$  are considered to be zero mean independent identically distributed (i.i.d.) Gaussian variables. The covariance matrix of the transmitted signal is given by:

$$R_{xx} = E\{XX^H\} \quad (1)$$

where  $E\{\cdot\}$  denotes the expectation and the operator  $X^H$  denotes the Hermitian of matrix  $X$ , which means the transpose and component-wise complex conjugate of  $X$ .

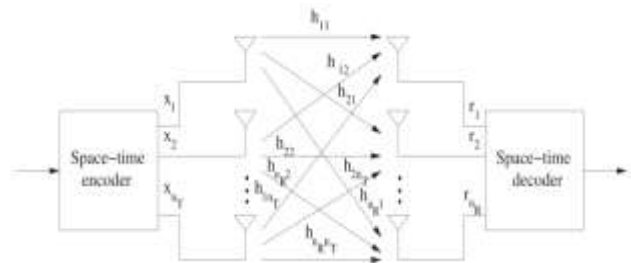


Figure 2: Block diagram of a MIMO system

The total transmitted power is constrained to  $P$ , regardless of the number of transmit antennas  $n_T$ . It can be represented as:

$$P = tr(R_{xx}) \quad (2)$$

Where  $tr(R_{XX})$  denotes the trace of matrix  $R_{XX}$ , obtained as the sum of the diagonal elements of  $R_{XX}$ . If the channel is unknown at the transmitter, it will be assumed that the signals transmitted from individual antenna elements have equal powers of  $P/n_T$ . The covariance matrix of the transmitted signal is given by

$$P_{xx} = \frac{P}{n_T} I_{n_T} \quad (3)$$

where  $I_{n_T}$  is the  $n_T \times n_T$  identity matrix. The transmitted signal bandwidth is narrow enough, so its frequency response can be considered as flat (a memoryless channel). The channel is described by an  $n_R \times n_T$  complex matrix, denoted by  $H$ . The  $ij$ -th component of the matrix  $H$ , denoted by  $h_{ij}$ , represents the channel fading coefficient from  $j$ -th transmit to the  $i$ th receive antenna. For normalization purposes it is assumed that the received power for each of  $n_R$  receive branches is equal to the total transmitted power. that is to say, signal attenuations and amplifications in the propagation process were ignored, including shadowing, antenna gains etc. Thus, the normalization constraint for the elements of  $H$ , on a channel with fixed coefficients is obtained as:

$$\sum_{j=1}^{n_T} h_{ij} = n_T, \quad i = 1, 2, 3, \dots, n_T \quad (4)$$

When the channel matrix elements are random variables, the normalization will apply to the expected value of the above expression. It is assumed that the channel matrix is known to the receiver, but not always at the transmitter. The channel matrix can be estimated at the receiver by transmitting a training sequence. The estimated channel state information (CSI) can be communicated to the transmitter via a reliable feedback channel. Assuming that the transmit power from each antenna in the equivalent MIMO channel model is  $P/n_T$ , the overall channel capacity  $C$ , can be estimated by using the Shannon capacity formula [7].

$$C = W \sum_{i=1}^r \log_2 \left( 1 + \frac{P_{ri}}{\delta^2} \right) \quad (5)$$

where  $W$  is the bandwidth of each sub-channel and  $P_{ri}$  is the received signal power in the  $i$ th sub-channel. It is given by

$$P_i = \frac{\lambda_i P}{n_T \delta^2} \quad (6)$$

Where  $\sqrt{\lambda_i}$  is the singular value of channel matrix  $H$ . Thus the channel capacity can be written as

$$C = W \sum_{i=1}^r \log_2 \left( 1 + \frac{\lambda_i P}{n_T \delta^2} \right) \quad (7)$$

### 3. MIMO CAPACITY FOR CHANNELS WITH FIXED COEFFICIENTS

The maximum possible transmission rates in a number of various channel settings were examined in this section. First the study focused on channels with constant matrix elements. In this case, the channel is assumed to be known only at the receiver, but not at the transmitter.

#### 3.1 Single Antenna Channel

In this scenario a channel with  $n_T = n_R = 1$  and  $H = h = 1$  is considered, the Shannon formula gives the capacity of this channel as:

$$C = W \log_2 \left( 1 + \frac{P_{ri}}{\delta^2} \right) = W \log_2 \left( 1 + \frac{P}{\delta^2} \right) \quad (8)$$

Where in this case  $P_{ri} = P$ ,  $n_T$  and  $n_R$  are the number of transmit and received antenna respectively and  $W$  is the bandwidth of the system. For high SNRs, the capacity grows logarithmically with the SNR. By assuming that the channel coefficient is normalized so that  $|h|^2 = 1$ , and for the SNR ( $P/\sigma^2$ ) of 20 dB, the capacity of a single antenna link is 6.658 bits/s/Hz.

#### 3.2 Coherent Combining

In this channel, with the channel matrix  $h_{ij}$ , the same signal is transmitted simultaneously from  $n_T$  antennas as proposed in [2], the received signal at antenna  $i$  is given by  $r_i = n_T x$  and the received signal power at antenna  $i$  is given by

$$P_{ri} = n_T^2 \frac{P}{n_T} = n_T P \quad (9)$$

where  $P/n_T$  is the power transmitted from one antenna and the total received power per receiving antenna is

$n_T P$ . The power gain of  $n_T$  in the total received power comes due to coherent combining of the transmitted signals.

The rank of channel matrix H is 1, and there is only one received signal in the equivalent channel model with the power

$$P_{ri} = n_T n_R P \quad (10)$$

Then from Shannon channel capacity, the capacity C is given by:

$$C = W \log_2 \left( 1 + n_T n_R \frac{P}{\delta^2} \right) \quad (11)$$

This system achieves a diversity gain of  $n_R \times n_T$  relative to a single antenna link. The cost of this gain is the system complexity required to implement coordinated transmissions and coherent maximum ratio combining. However, the capacity grows logarithmically with the total number of antennas  $n_R \times n_T$ . If  $n_T = n_R = 5$  and  $10 \log_{10} P/\sigma^2 = 20$  dB, the normalized capacity  $C/W$  is 11.28 bits/sec/Hz as against 6.658 bits/sec/Hz achieved with single antenna.

### 3.3 Non-coherent Combining

In this case, the signals transmitted from various antennas are taken as different and all channel entries are equal to 1, there is only one received signal in the equivalent channel model with the power  $P_{ri} = n_R P$ . Thus by proper substitution in the Shannon channel capacity formula, the capacity is given by (Zhu, 2010):

$$C = W \log_2 \left( 1 + n_R \frac{P}{\delta^2} \right) \quad (12)$$

For the same SNR of 20 dB and  $n_R = n_T = 5$  as in the case of coherent combining, the capacity in this case is 8.96 bits/sec/Hz.

## 4. MATERIAL AND METHOD

The capacity for different communication systems were modelled using MATLAB. The program uses a well-known mathematical model based on Shannon capacity model given by equation (11). Where the SNR on each link after a Singular Value Decomposition (SVD) has been done at the receiver according to [9].

### 4.1 Models Developed

SIMULINK blocks of MATLAB® (Release 2008b) was used to developed the communication model for a single channel to further investigate the Bit Error Rate (BER) of the system as function of the  $E_b/N_0$  which is given by (13).

$$BER = \left( \frac{1}{2} \right) \operatorname{erfc} \left( \frac{E_b}{N_0} \right) \quad (13)$$

Where:

$N_0$  = Noise power spectral density (W/Hz).

$E_b/N_0$  is the energy per bit to noise power spectral density ratio.

$\operatorname{erfc}$  is the error function.

In this model, the signal generated in the random signal generator goes into the transmitter and modulated. The output of the transmitter (modulated signal) passes over the channel which is characterized by multipath Raleigh fading. Some form of errors are introduced into the signal due to channel impairment. This signal is then passed into the receiver which demodulate it and pass it to an error rate calculation block. This block compares this Signal with the original signal from the random integer generator and calculate the total errors and the error rate for a total symbol simulated.

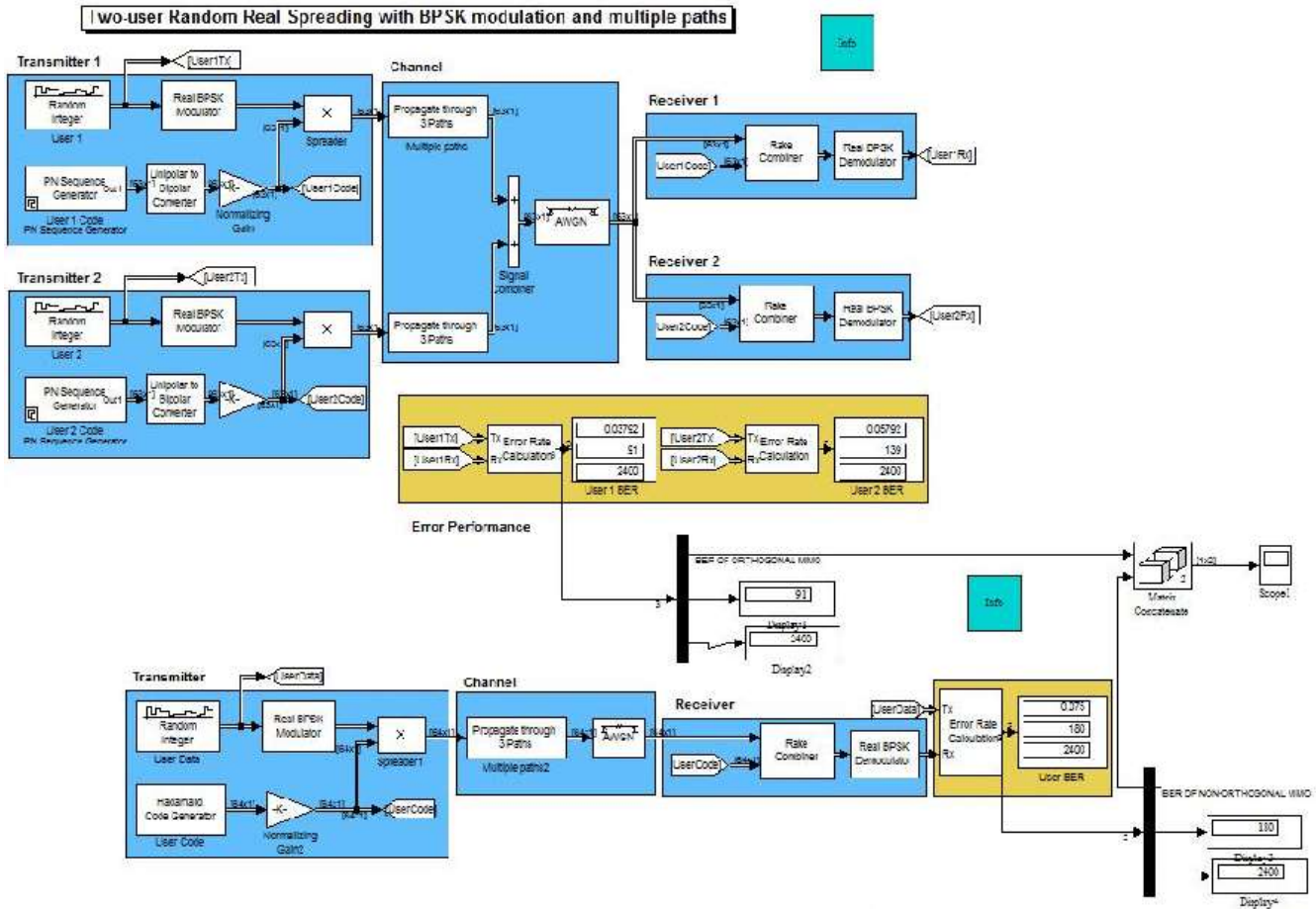


Figure 3: MATLAB SIMULINK blocks for the representative model for Single and Multi-user with and without Orthogonal Transmission

**5. RESULTS**

The results obtained are presented in table 1 and plots 1 to 3 and the metric that was used to obtain comparison measured among the Capacity of different systems is in percentage (%). These values are calculated by using the following percentage expression:

$$\begin{aligned} \text{Result \%} &= \left( \frac{\text{Capacity of A} - \text{Capacity of B}}{\text{Capacity of B}} \right) * 100\% \\ &= \left( \frac{\text{Capacity of A}}{\text{Capacity of B}} - 1 \right) * 100\% \end{aligned}$$

This means that for example 20% is equivalent to saying that capacity of A is 0.2 times higher than capacity of B, or equivalently, capacity of A is equal to 1.2 times that of B. Similarly, 200% means that capacity of A is an increase of 2 times the capacity of

B (with respect to capacity of B) or equivalently 3 times higher than B. Thus, although from the tables and the plots, the results are shown as the capacity increase, but the remark will be given as the number of times that capacity of A is higher than B.

Figure 4.0 illustrates the plot of capacity (bit/Sec/Hz) against the number of antennas for non-orthogonal MIMO Channel. It shows that, the capacity of the system increases sharply at lower number of antenna. As the number of antenna increases, it saturates to a value which in this case was found to be 10<sup>2</sup> bit/Sec/Hz as shown in the Figure.

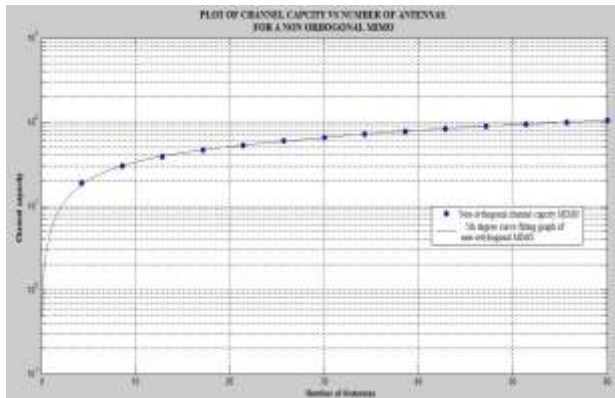


Figure 4: Plot of Capacity (bit/Sec/Hz) vs. Number of antennas for Non-orthogonal MIMO Channel

In similar way, Figure 5.0 illustrates the plot of capacity (bit/Sec/Hz) against the number of antennas for non-orthogonal MIMO Channel. It shows that, the capacity of the system increases sharply at lower number of antennas. As the number of antennas increases, it saturates to a value which was found to lie in between  $10^3$  bit/Sec/Hz and  $10^4$  bit/Sec/Hz as shown.

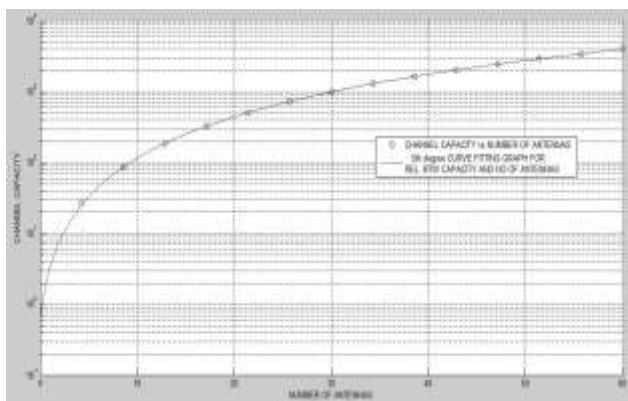


Figure 5: Plot of capacity (bit/sec/Hz) vs. number of antennas for orthogonal MIMO Channel.

Figure 6 illustrates the plot of capacity (bit/Sec/Hz) against the number of antennas for both orthogonal and non-orthogonal MIMO Channel. It shows that, the capacity of the system increases sharply at lower number of antennas as the number of antennas increases.

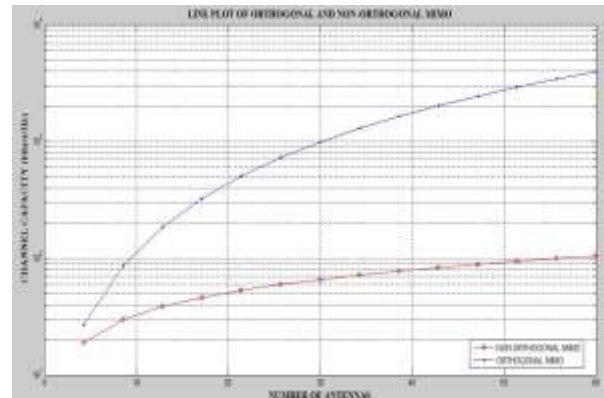


Figure 6: Plot of capacity (bit/sec/Hz) vs. number of antennas comparing orthogonal and non-orthogonal MIMO channels

Figure 7.0 is a Bar chart representation of Figure 4.8 which illustrates the plot of capacity (bit/Sec/Hz) against the number of antennas for both orthogonal and non-orthogonal MIMO Channel. It shows that, the capacity of the system increases sharply at lower number of antennas. As the number of antennas increases, it saturates to a constant value which can be determined by differentiating equation 2.19 and equation 2.20 and set to zero.

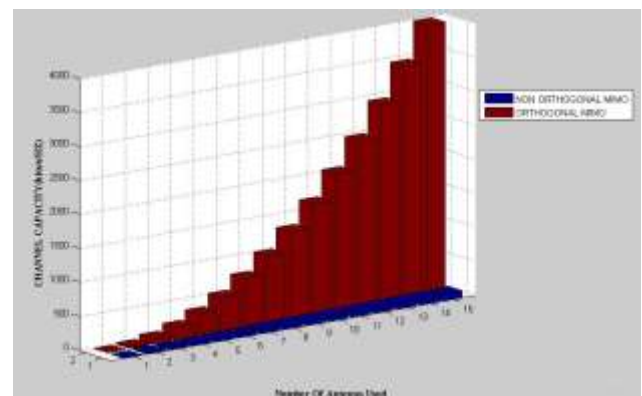


Figure 7: Bar chart showing Capacity (bit/Sec/Hz) vs. number of antennas comparison for orthogonal and non-orthogonal MIMO channels.

From the plots of Figure 4.0 through Figure 7.0, it can observe that, as the number of antennas increases, the Channel Capacity for both system increase. The results obtained from comparison of the three different Capacities in relation to single channel shows that, the Capacity obtained from Coherent channel is 1.246 higher than that of single channel. Also, that of non-

coherent channel is 1.122 times that of single channel. Using the same index, Orthogonal channel gives an average capacity of  $n/2$  times higher than that of single antenna, this implies that, for a given number of antenna say Eight (8) at the same signal to noise ratio, the capacity of orthogonal channel will be about four (4) times that of single channel. The Capacity obtained from the orthogonal system grows faster than that of the non-Orthogonal system most especially at higher signal to noise ratio. Even among the MIMO systems, when the information is being sent orthogonally where all the channels are parallel to each other and are not correlated, the probability of having all the information turning out to be bad due to deep fading is very low. This will therefore make its capacity higher than the other non-orthogonal cases.

## 6. CONCLUSION

Comparison of the three different Capacities in relation to a system with single channel revealed that, the Capacity obtained from coherent channel is 1.246 higher than that of single channel. Also, that of non-coherent channel is 1.122 times that of single channel. Using the same index, Orthogonal channel gives an average capacity of  $n/2$  higher than that of single antenna.

In summary, the research serve as a valuable tool as it exploit the advantages of using MIMO technique towards increasing higher data rate, better coverage, improved spectrum efficiency and reduced operating cost which could be useful both to telecommunications service users, as a basis for understanding and assessing possible differences between competing services, and to service providers, as a means of determining what improvements in service performance are needed to assure customer satisfaction.

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